Job Market Paper

Idiosyncratic Volatility, Growth Options, and the Cross-Section of Returns[†]

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Abstract

The paper presents a simple real options model that explains why in cross-section high idiosyncratic volatility implies low future returns and why the value effect is stronger for high volatility firms. In the model, high idiosyncratic volatility makes growth options a hedge against aggregate volatility risk. Growth options become less sensitive to the underlying asset value as idiosyncratic volatility goes up. It cuts their betas and saves them from losses in volatile times that are usually recessions. Growth options value also positively depends on volatility. It makes them a natural hedge against volatility increases. In empirical tests, the aggregate volatility risk factor explains the idiosyncratic volatility discount and why it is stronger for growth firms. The aggregate volatility risk factor also partly explains the stronger value effect for high volatility firms. I also find that high volatility and growth firms have much lower betas in recessions than in booms.

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1 Introduction

A recent paper by Ang, Hodrick, Xing, and Zhang (2006) (hereafter - AHXZ) finds that firms with high idiosyncratic volatility earn negative abnormal returns. The return differential between high and low volatility firms is around 13% per year. Meanwhile, the conventional wisdom says that, if anything, the relation between idiosyncratic volatility and future returns should be positive. In what follows, I call this puzzle the idiosyncratic volatility discount.

Another recent paper by Ali, Hwang, and Trombley (2003) finds that the value effect is about 6% per year larger for high idiosyncratic volatility firms. It poses a challenge to any risk-based story for the value effect. Any such story has to explain why the value effect is related to something that is seemingly not risk - idiosyncratic volatility.

My paper develops a real options model that provides a risk-based explanation for both puzzles. In my model, higher idiosyncratic volatility makes growth options less sensitive to the current value of the underlying asset. The beta of the underlying asset does not change with idiosyncratic volatility, so the response of the underlying asset value to a given market return stays the same. However, the lower growth options sensitivity to the value of the underlying asset means that the response of the growth options value to the same market return decreases with idiosyncratic volatility. That is, higher idiosyncratic volatility means lower beta of growth options.

My model also suggests a new macroeconomic hedging channel. In recessions, both aggregate volatility and idiosyncratic volatility increase[1]. The increase in idiosyncratic volatility makes growth options betas smaller and mutes the increase in their risk premiums. Because a lower expected return means a higher current price, the value of growth options drops less as the bad news arrives if the idiosyncratic volatility of the underlying asset is higher.

Higher volatility in bad times also means higher value of growth options. Hence, aggregate volatility increases in recessions mean higher returns for growth firms than for value firms. My model shows that this effect is also stronger for high volatility firms.

These two effects form what I call the idiosyncratic volatility hedging channel. In my model, this channel is stronger for high idiosyncratic volatility firms, which makes them

[1] See, e.g., Campbell, Lettau, Malkiel, and Xu, 2001

good hedges against adverse business cycle shocks. The second part of the idiosyncratic volatility channel can also contribute to our understanding of why value firms are riskier than growth firms.

The idiosyncratic volatility hedging channel works through economy-wide changes in volatility. Therefore, I link it to the concept of aggregate volatility risk developed in Campbell (1993) and Chen (2002). The models in Campbell (1993) and Chen (2002) are the extensions of Merton (1973) Intertemporal CAPM (henceforth ICAPM). In the Campbell model, higher aggregate volatility implies higher future risk premium. The stocks that covary negatively with changes in aggregate volatility command a risk premium, because they lose value when the future is also turning bleak.

In the Chen model, investors care not only about future returns, but also about future volatility. Aggregate volatility increases imply the need to boost precautionary savings and to cut current consumption. The stocks that covary negatively with aggregate volatility changes again command a risk premium, but for a different reason. They lose value exactly when consumption is reduced to build up savings.

The pricing of aggregate volatility risk is empirically confirmed by AHXZ in the same paper that establishes the idiosyncratic volatility discount. The return differential between the firms with the most and the least negative covariance with expected aggregate volatility changes is about 12% per year.

The stronger idiosyncratic volatility hedging channel for high idiosyncratic volatility firms implies that these firms have the lowest exposure to aggregate volatility risk. Their expected returns increase the least and their prices drop the least as expected aggregate volatility goes up and a recession begins. Therefore, high volatility stocks provide additional consumption when future prospects become worse and the need for precautionary savings increases.

The value effect is, by definition, the return differential between growth options firms and assets in place firms. In my model, idiosyncratic volatility diminishes the growth options market beta and their exposure to aggregate volatility risk, but has no impact on assets in place. Hence, the expected return differential between value firms and growth firms should be wider for high volatility firms. The new testable hypothesis is that the stronger value effect for high volatility firms can be explained by aggregate volatility risk. The important implication is that aggregate volatility risk partly explains the value effect.

In my model, the idiosyncratic volatility discount is created by the change in the risk of growth options. The larger is the relative value of growth options in the firm value, the higher is the impact of idiosyncratic volatility on the firm's risk. The new empirical hypothesis is that the idiosyncratic volatility discount is stronger for growth firms. The other empirical hypothesis is that aggregate volatility risk explains the difference in the idiosyncratic volatility discount between growth and value firms.

I start empirical tests by sorting firms on market-to-book and idiosyncratic volatility. As the model predicts, the idiosyncratic volatility discount starts at zero for value firms and monotonically increases with market-to-book.

I also run cross-sectional regressions of firm returns on lagged firm characteristics. In the cross-sectional regressions, the product of market-to-book and idiosyncratic volatility is negative and strongly significant. Adding the product flips the signs of idiosyncratic volatility and market-to-book. The first sign change confirms that the idiosyncratic volatility discount is absent for low market-to-book (value) firms and increases with market-to-book. The second sign change shows that the value effect is absent for low volatility firms and suggests that my model can potentially explain the observed part of the value effect.

I also find that controlling for idiosyncratic volatility in the cross-sectional regressions increases the size effect by about a half and makes it much more significant. It is quite intuitive, because small firms are usually high idiosyncratic volatility firms. The size effect predicts high returns to these firms, and the idiosyncratic volatility discount predicts just the opposite. Hence, not controlling for either of them weakens the estimate of the other.

In time-series tests, I use the ICAPM to explain the idiosyncratic volatility discount, the stronger idiosyncratic volatility discount for growth firms, and the stronger value effect for high volatility firms. To test the prediction of my model that the three idiosyncratic volatility effects are explained by aggregate volatility risk, I introduce an aggregate volatility risk factor similar to the one in AHXZ. I call it the BVIX factor. The BVIX factor is based on stock return sensitivity to changes in the CBOE VIX index. The VIX index measures the implied volatility of S&P 100 options. AHXZ show that changes in VIX are a good proxy for changes in expected aggregate volatility. I define the BVIX factor as the zero-cost portfolio long in firms with the most negative and short in firms with the most positive return sensitivity to changes in VIX.

I find that high volatility firms, growth firms, and especially high volatility growth firms have negative BVIX betas. Their BVIX betas are also significantly lower than the betas of low volatility, value, and low volatility value firms. It means that high volatility, growth, and especially high volatility growth firms are good hedges against aggregate volatility risk. Their value goes up when aggregate volatility increases and most stocks witness negative returns.

The ICAPM with the BVIX factor completely explains the idiosyncratic volatility discount and why it is stronger for growth firms. The BVIX factor also reduces the strong value effect for high volatility firms by about a third. I also corroborate the BVIX results by showing that conditional market betas of high volatility, growth, and especially high volatility growth firms are lower in recessions than in booms.

The BVIX factor has a broader use than explaining the effects of idiosyncratic volatility on returns. I show that the BVIX factor is priced for several portfolio sets. The ICAPM with BVIX successfully competes with the Fama-French model. In Barinov (2007a), I also show that the BVIX factor explains the low returns to small growth firms, IPOs, and SEOs, which are the worst failures of the existing asset-pricing models.

The Merton (1987) model predicts a positive relation between idiosyncratic volatility and expected returns for risky assets. It does not contradict my model that predicts the opposite relation for common stock. Rather, my model emphasizes the option-like nature of common stocks, which produces another effect in the opposite direction. Therefore, my model is consistent with the evidence supporting the Merton model for other risky assets[2].

My model is related to Veronesi (2000) and Johnson (2004). They show that parameter risk can negatively affect expected returns by lowering the covariance with the stochastic discount factor. Johnson (2004) also uses the idea that the beta of a call option is negatively related to volatility. In my paper, I take a broader definition of idiosyncratic risk. I show that it can affect expected returns even if there is no parameter risk. I also focus on growth options instead of focusing on leverage, as Johnson (2004) does. It allows me to study the relation between idiosyncratic volatility and the value effect.

My cross-sectional results in the empirical part are close to Ali, Hwang, and Trombley (2003). Ali et al. (2003) argue that idiosyncratic volatility is a proxy for limits to arbitrage and therefore the value effect should be stronger for high volatility firms. However, Ali et

[2]Green and Rydqvist (1997) find a positive relation between idiosyncratic risk and expected returns for lottery bonds. Bessembinder (1992) and Mansi, Maxwell, and Miller (2005) find a similar relation for currency and commodity futures and corporate bonds, respectively.

al. (2003) do not study the implications of this fact for the idiosyncratic volatility discount. They also fail to find that controlling for the product of market-to-book and idiosyncratic volatility in Fama-MacBeth regressions flips the signs of market-to-book and volatility.

The other empirical study close to my paper is AHXZ, which is the first to establish the idiosyncratic volatility discount and the pricing of aggregate volatility risk. My paper extends AHXZ by showing both theoretically and empirically that the idiosyncratic volatility discount is explained by aggregate volatility risk. I also extend AHXZ by linking the idiosyncratic volatility discount and aggregate volatility risk to growth options.

Nagel (2004) and Boehme, Danielsen, Kumar, and Sorescu (2006) find that the idiosyncratic volatility discount is higher if limits to arbitrage are high. My result that the idiosyncratic volatility discount exists only for growth stocks is distinct from theirs. In cross-sectional regressions, controlling for the product of limits to arbitrage and idiosyncratic volatility does not subsume the product of market-to-book and idiosyncratic volatility. In portfolio sorts, the link between the idiosyncratic volatility discount and limits to arbitrage disappears after I control for the known risk factors.

Several empirical studies (e.g., Malkiel and Xu, 2003) find positive relation between idiosyncratic volatility and future stock returns at the portfolio level. This evidence is not inconsistent with my model that studies the same relation at the firm level. Firm-level idiosyncratic volatility is diversified away at the portfolio level. The remaining portfolio-level idiosyncratic volatility is more likely to result from omitted common factors. Hence, the two idiosyncratic volatility measures are likely to be poor proxies for each other.

The possible applications of the ideas in the paper stretch far beyond explaining the idiosyncratic volatility discount. I show that high idiosyncratic volatility creates a hedge against aggregate volatility risk and means lower expected returns. Therefore, more information and less uncertainty about a firm can hurt, if it comes in the wrong place. The wrong place is any asset behind a valuable real option. This idea has important implications for the studies of the link between firm value and expected return on the one hand and information quality, accounting quality, disclosure, etc., on the other.

In addition, establishing the link between idiosyncratic volatility and risk opens the gate to rethinking the results of the studies that use idiosyncratic volatility as a proxy for limits to arbitrage. Abundant evidence that many anomalies are stronger for high volatility firms can mean that the anomalies are related to aggregate volatility risk.

The paper proceeds as follows. Section 2 lays out the model and derives its empirical implications. Section 3 discusses the data sources and shows descriptive statistics. Section 4 and Section 5 test the cross-sectional and time-series implications of my model. Section 6 discusses the robustness of the idiosyncratic volatility discount and tests the competing behavioral stories. Section 7 offers the conclusion. The proofs of the propositions in text are collected in Appendix.

2 The Model

2.1 Cross-Sectional Effects

Consider a firm that consists of growth options, P_t, and assets in place, B_t. The growth options are represented by a European call option, which gives the right to receive at time T S_T for price K. Both S_t, the price of the asset underlying the growth options, and B_t follow geometric Brownian motions:

$$dS_t = \mu_S S_t dt + \sigma_S S_t dW_S + \sigma_I S_t dW_I \tag{1}$$

$$dB_t = \mu_B B_t dt + \sigma_B B_t dW_B \tag{2}$$

The stochastic discount factor process is given by

$$d\Lambda_t = -r\Lambda_t dt + \sigma_\Lambda \Lambda_t dW_\Lambda \tag{3}$$

dW_I is the purely idiosyncratic component of S_t and is assumed to be uncorrelated with the pricing kernel and, for simplicity, with W_S and W_B, though relaxing the second assumption will not change the results. I also assume for simplicity that there is no purely idiosyncratic component in B_t (relaxing this assumption also does not change anything).

dW_I represents firm-specific shocks to growth options value. While the part of dW_S that is orthogonal to the pricing kernel is also firm-specific, I need dW_I to be able to increase the variance of the firm-specific shocks without increasing the covariance of S_t with the pricing kernel.

I do not assume anything about the correlation between W_S and W_B. The underlying asset of growth options and assets in place in my model are driven by two different processes, but these processes can be highly correlated.

The no-arbitrage condition and the definition of the pricing kernel imply that

$$dB_t = (r + \pi_B)B_t dt + \sigma_B B_t dW_B \tag{4}$$

$$dS_t = (r + \pi_S)S_t dt + \sigma_S S_t dW_S + \sigma_I S_t dW_I \tag{5}$$

where $\pi_B = -\rho_{B\Lambda}\sigma_B\sigma_\Lambda$ and $\pi_S = -\rho_{S\Lambda}\sigma_S\sigma_\Lambda$ are the risk premiums. The idiosyncratic risk is not priced for the unlevered claim on the asset behind growth options and it will not be priced for assets in place if I assume that they also carry some purely idiosyncratic risk.

However, for growth options the idiosyncratic risk is priced:

Proposition 1. The value of the firm is given by

$$dV_t/V_t = (r + \pi_B - (\pi_B - \pi_S\Phi(d_1)\frac{S_t}{P_t}) \cdot \frac{P_t}{V_t})dt + \Phi(d_1)\frac{S_t}{V_t}(\sigma_S dW_S + \sigma_I dW_I) + \sigma_B\frac{B_t}{V_t}dW_B \tag{6}$$

$$\text{where} \quad d_1 = \frac{log(S/K) + (r + \sigma_S^2/2 + \sigma_I^2/2)(T-t)}{\sqrt{(\sigma_S^2 + \sigma_I^2) \cdot (T-t)}} \tag{7}$$

If assets in place are riskier than growth options, $\pi_B - \pi_S\Phi(d_1)S_t/P_t > 0$, then the expected rate of return to the firm (the drift in the firm value, μ_V) decreases in idiosyncratic risk, σ_I, and increases in the value of assets in place, B.

Proof: See Appendix A.

The intuition of the proof is that the idiosyncratic risk discount consists of two parts and relies on the existence of the value effect. First, an increase in idiosyncratic risk reduces the expected return by reducing elasticity of the growth options value with respect to the underlying asset value ($\Phi(d_1)S_t/P_t$). Second, an increase in idiosyncratic risk increases the relative value of growth options (P_t/V_t) and makes the firm more growth-like, which decreases expected returns if the value effect exists.

By definition, the beta of the option is determined by, first, how responsive the underlying asset is to a percentage change in the risk factor and, second, how responsive the price of the option is to a percentage change in the price of the underlying asset. Hence, the beta of the option is equal to the product of the elasticity and the beta of the underlying asset. The elasticity decreases as volatility increases because if volatility is high, a change in the underlying asset price is less informative about its value at the expiration date. When idiosyncratic volatility goes up, the elasticity declines and the beta of the underlying asset stays constant, hence their product - the beta of growth options - decreases.

The idiosyncratic risk in my model is idiosyncratic at the level of the underlying assets, but its presence changes the systematic risk of growth options. If one pools the underlying assets, the risk will be diversified away, and this is the reason it is not priced for the unlevered claim on any of them. However, if one pools the underlying assets and then creates an option on them, the decrease in the idiosyncratic volatility will lead to the systematic risk of the option being greater than the systematic risk of the portfolio of separate options on each of the underlying assets.

The proof of Proposition 1 in Appendix A shows that in the current setup the sufficient (though not necessary) condition for the existence of the idiosyncratic volatility discount is that assets in place are riskier than growth options. There are currently two strands of the value effect literature that make this prediction. A good example of the first strand is Zhang (2005) that argues that assets in place are riskier in recessions because of costly divesture. The second strand starts with Campbell and Vuolteenaho (2004) that shows that value firms have higher cash flow betas and growth firms have low cash flow betas, and the cash flow risk earns a much higher risk premium.

In Section 2.2 I also provide a new explanation of why growth options earn lower return than assets in place. The main idea there is that the volatility increase in the recession makes growth options more valuable. Holding all other effects fixed, the value of growth options is therefore less negatively correlated with aggregate volatility. Growth options provide additional consumption when expected aggregate volatility is high and, consequentially, future investment opportunities are worse and the need for precautionary savings is higher. It makes growth options more desirable and their expected returns lower. In this subsection, however, I just assume a low risk premium for the underlying asset of growth options to keep things simple.

Corollary 1. Define $IVar$ as the variance of the part of the return generating process (6), which is orthogonal to the pricing kernel. Then the idiosyncratic variance $IVar$ is

$$IVar = \sigma_S^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} \cdot (1 - \rho_{S\Lambda}^2) + \sigma_B^2 \cdot \frac{B^2}{V^2} \cdot (1 - \rho_{B\Lambda}^2) + \\ + \sigma_I^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} + \sigma_S \cdot \sigma_B \cdot \Phi(d_1) \cdot \frac{S}{V} \cdot \frac{B}{V} \cdot (\rho_{SB} - \rho_{B\Lambda} \cdot \rho_{S\Lambda}) \tag{8}$$

I show that for all reasonable parameter values σ_I

$$\frac{\partial IVar}{\partial \sigma_I} > 0, \tag{9}$$

which implies that my empirical measure of idiosyncratic volatility - the standard deviation of Fama-French model residuals - is a noisy but valid proxy for σ_I.

Proof: See Appendix A.

Corollary 1 shows that the idiosyncratic volatility depends positively on the idiosyncratic risk parameter. It is also impacted by some other factors, which means that it is a valid, although noisy, proxy for the idiosyncratic risk parameter. I do not claim that idiosyncratic volatility is the best proxy for idiosyncratic risk. All I need to tie my model to the data is that it is positively correlated with idiosyncratic risk, and Corollary 1 shows that it should be true.

Leaning on Corollary 1, in the rest of the section I use the terms "idiosyncratic volatility" and "idiosyncratic risk" interchangeably.

Corollary 2. The expected return differential between assets in place and growth options, $\pi_B - \pi_S \Phi(d_1) S_t / P_t$, is increasing in idiosyncratic risk.

Proof: Follows from the well-known fact that the option price elasticity with respect to the price of the underlying asset, $\Phi(d_1) S_t / P_t$, is decreasing in volatility.

Corollary 2 suggests a simple reason why in the rational world the value effect is higher for high volatility firms, as Ali et al. (2003) show. High volatility reduces the expected returns to growth options by reducing their elasticity with respect to the value of the underlying asset (and therefore reducing their beta) and leaves assets in place unaffected.

Corollary 2 implies that the observed value effect can wholly be an idiosyncratic volatility phenomenon. The return differential between growth options and assets in place can take different signs at different levels of idiosyncratic volatility. If the value effect is actually negative at zero idiosyncratic volatility, and positive at the majority of its empirically plausible values, the value effect will be on average positive even though growth options are inherently (absent idiosyncratic volatility) riskier than assets in place. In this case, the observed part of the value effect will be created only by the interaction between idiosyncratic volatility and growth options captured by my model.

Proposition 2. The effect of idiosyncratic volatility on returns, $\left| \frac{\partial \mu_V}{\partial \sigma_I} \right|$, is decreasing in the value of assets in place, B.

Proof: See Appendix A.

The main idea behind Proposition 2 is that without growth options or with very large B_t idiosyncratic volatility will not have any impact on returns. As growth options take a greater fraction of the firm, the impact of idiosyncratic volatility on returns becomes stronger, since it works through growth options. Also, more idiosyncratic volatility makes growth options less risky, while the risk of assets in place stays constant. It means a wider expected return spread between growth options and assets in place. The positive cross-derivative captures both effects.

The sign of the excess return derivative in Proposition 2 implies that in the cross-sectional regression the product of market-to-book and volatility is negatively related to future returns. In portfolio sorts Proposition 2 predicts large and significant idiosyncratic volatility discount for growth firms and no idiosyncratic volatility discount for value firms. Proposition 2 also predicts stronger value effect for high volatility firms.

Hypothesis 1. The cross-sectional regression implied by my model is

$$Ret \approx a - b \cdot M/B + c \cdot (M/B)_0 \cdot IVol - c \cdot M/B \cdot IVol + \delta Z, \; a, \; c > 0 \qquad (10)$$

where $(M/B)_0$ is the market-to-book ratio for the firm with no growth options and Z are other priced characteristics.

It implies that

$$\frac{\partial Ret}{\partial M/B} \approx -b - c \cdot IVol < 0 \qquad (11)$$

$$\frac{\partial Ret}{\partial IVol} \approx -c \cdot (M/B - (M/B)_0) < 0 \qquad (12)$$

I predict that in cross-sectional regressions the coefficient of idiosyncratic volatility, $c \cdot (M/B)_0$, is positive. The coefficient of the volatility product with market-to-book, c, is negative. The ratio of the coefficients equals to $(M/B)_0$, the market-to-book of the firm with no growth options. For the firm with no growth options, as (12) shows, the two terms cancel out and idiosyncratic volatility has no impact on returns. While the lowest possible market-to-book is 1 in my model, in Hypothesis 1 I replace 1 with an unknown $(M/B)_0$. $(M/B)_0$ is likely to be lower than 1, because book values lag market values and losses in the market value may be unrecognized in the book value for some time.

Equation (11) divides the observed value effect into two parts. The first one is denoted by b and represents the part of the value effect, which is unrelated to idiosyncratic volatility and comes from the difference in expected returns to assets in place and growth options absent idiosyncratic volatility. The second one is denoted $c \cdot IVol$ and represents the part of the value effect, which is driven by the interaction between growth options and idiosyncratic volatility. My model makes no prediction about the magnitude of the first part and even its sign.

The theoretical results in this section rely on the fact that growth options are call options on the projects behind them. In theory, any option-like dimension of the firm can be used to generate similar results, i.e. the idiosyncratic volatility discount that increases as the firm becomes more option-like. One well-known option-like dimension of the firm is leverage, which can replace growth options in the discussion above.

The motivation of looking at market-to-book rather than leverage is two-fold. First, using market-to-book in my model helps to explain the puzzling increase of the value effect with idiosyncratic volatility. The explanation will contribute to our understanding of the value effect. Second, the effects of idiosyncratic volatility on expected returns are stronger if the call option is closer to being in the money. For example, holding the value of growth options fixed, several at-the-money projects create stronger idiosyncratic volatility effects than one deep-in-the-money project. The call option created by leverage is at the money when the firm is close to bankruptcy. Hence, growth options are usually closer to being at the money than the call option created by leverage. So, I expect growth options to be more important in understanding the idiosyncratic volatility discount.

Empirically, market-to-book and leverage are strongly inversely related. One reason is the mechanical correlation created by the market value being in the numerator of market-to-book and in the denominator of leverage. There are also several corporate finance theories predicting that growth firms should choose lower leverage (e.g., the free cash flow problem). Hence, in empirical tests the possible link between the idiosyncratic volatility discount and leverage should work against finding any relation between the idiosyncratic volatility discount and market-to-book.

2.2 The Idiosyncratic Volatility Hedging Channel

In the previous subsection I developed predictions about the impact of idiosyncratic volatility on the cross-section of returns. I derived from my model the three idiosyncratic volatility effects: the idiosyncratic volatility discount, the stronger idiosyncratic volatility discount for growth firms, and the higher value effect for high volatility firms. In this subsection, I sketch the ICAPM-type explanation of why the link between idiosyncratic volatility and expected returns cannot be captured by one-period models.

Campbell (1993) develops a model of aggregate volatility risk, where aggregate volatility increase means higher future risk premium. In Campbell (1993) the assets that react less negatively to aggregate volatility increases, offer an important hedge against adverse business-cycle shocks. These stocks earn a lower risk premium, because they provide consumption when future investment opportunities become worse.

Chen (2002) develops a model offering another reason why the assets that react less negatively to aggregate volatility increases can be valuable. In his model, investor care not only about future investment opportunities, but also about future volatility. An increase in expected aggregate volatility means the need to reduce current consumption in order to build up precautionary savings. The stocks that do not go down as aggregate volatility goes up provide consumption when it is most needed and therefore earn a lower risk premium.

My model goes further by predicting what types of firms will have the lowest, probably negative, aggregate volatility risk. I show that the presence of idiosyncratic volatility and its close time-series correlation with aggregate volatility[3] creates the economy-wide idiosyncratic volatility hedging channel that consists of two parts. One part comes from the impact of idiosyncratic volatility on expected returns, and the other comes from the impact of idiosyncratic volatility on the value of growth options. This subsection shows that the idiosyncratic volatility hedging channel makes the prices of high volatility, growth, and high volatility growth firms covary least negatively with aggregate volatility, which means lower exposure to aggregate volatility risk.

In unreported findings I show that the idiosyncratic volatility of low and high volatility firms respond to aggregate volatility movements by changing by the same percentage rather than by the same amount. Therefore, the key variable in the time-series dimension is the elasticity of risk premium with respect to volatility, instead of the derivative, which was the focus of the cross-sectional analysis in the previous subsection.

Proposition 3 The elasticity of the risk premium in my model decreases (increases in the absolute magnitude) as idiosyncratic volatility increases:

$$\frac{\partial}{\partial \sigma_I}\left(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}\right) < 0 \tag{13}$$

[3] See Campbell, Lettau, Malkiel, and Xu, 2001, and Goyal and Santa-Clara, 2003

The elasticity of the risk premium in my model increases (decreases in the absolute magnitude) as the value of assets in place increases:

$$\frac{\partial}{\partial B}\left(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}\right) > 0 \tag{14}$$

The second cross-derivative of the elasticity with respect to idiosyncratic volatility and assets in place is positive:

$$\frac{\partial^2}{\partial \sigma_I \partial B}\left(\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}\right) > 0 \tag{15}$$

Proof: See Appendix A.

Proposition 3 summarizes the first part of the idiosyncratic volatility hedging channel. As aggregate volatility increases, the future risk premium and idiosyncratic volatility also increase. The previous subsection shows that high idiosyncratic volatility means lower risk and lower expected returns. By Proposition 3, for high volatility firms the future risk premium goes up less than for low volatility firms. The impact on current stock prices is exactly opposite, because higher expected return means lower current price, all else equal. So, Proposition 3 implies that the stock prices of high volatility firms will react less negatively to aggregate volatility increases than the stock prices of low volatility firms. The identical reasoning can be repeated for growth firms and high volatility growth firms.

A 50% increase and even a 100% increase in idiosyncratic volatility is not uncommon in recessions (see e.g., Figure 4 in Campbell, Lettau, Malkiel, and Xu, 2001). The simulations in the web appendix[4] show that the impact of such idiosyncratic volatility changes on the risk premium is substantial. In the simulations, the risk premium elasticity with respect to idiosyncratic volatility varies from zero for low volatility value firms to -0.5 for high volatility firms. It means that, net of any other effects of the recession on the risk premium, in recessions the idiosyncratic volatility hedging channel can reduce the expected returns to high volatility growth firms by a quarter or even a half.

Proposition 4 The elasticity of the firm value with respect to idiosyncratic volatility increases with idiosyncratic volatility:

$$\frac{\partial}{\partial \sigma_I}\left(\frac{\partial V}{\partial \sigma_I} \cdot \frac{\sigma_I}{V}\right) > 0 \tag{16}$$

The elasticity of the firm value decreases in the value of assets in place:

$$\frac{\partial}{\partial B}\left(\frac{\partial V}{\partial \sigma_I} \cdot \frac{\sigma_I}{V}\right) < 0 \tag{17}$$

[4] **http://outside2.simon.rochester.edu/phdresumes/barinov_alexander/simulations.pdf**

The second cross-derivative of the elasticity with respect to idiosyncratic volatility and assets in place is negative:

$$\frac{\partial^2}{\partial \sigma_I \partial B}(\frac{\partial V}{\partial \sigma_I} \cdot \frac{\sigma_I}{V}) < 0 \tag{18}$$

Proof: See Appendix A.

Proposition 4 summarizes the second part of the idiosyncratic volatility hedging channel. As the economy enters the recession and volatility increases, the value of growth options, like the value of any option, tends to increase with volatility. This hedging channel is naturally stronger for growth firms, because their return is more affected by the changes in the growth options value. This is a new explanation of why growth firms are less risky than value firms.

Based on simulations, I conclude that this hedging channel is also stronger for high volatility firms than for low volatility firms and that it is the strongest for high volatility growth firms. The simulations also show that the firm value elasticity with respect to idiosyncratic volatility is substantial. It varies from 0 for low volatility value firms to -0.3 and higher for high volatility growth firms. Therefore, net of any other cash flow effects of the recession, the increase in idiosyncratic volatility during the recession can increase the value of high volatility growth firms by 15-20%.

The bottom line of Propositions 3 and 4 is that high volatility, growth, and high volatility growth firms covary least negatively with changes in aggregate volatility. Hence, these three types of firms hedge against aggregate volatility risk. The reason is the idiosyncratic volatility channel, which predicts that the value of volatile growth options goes up the most as aggregate volatility and idiosyncratic volatility increase, and the expected risk premium of volatile growth options increases the least during volatile times.

Hypothesis 2. High idiosyncratic volatility firms, growth firms, and especially high idiosyncratic volatility firms hedge against aggregate volatility risk. Their betas with respect to the aggregate volatility risk factor are negative and lower than those of low volatility, value, and low volatility value firms.

The difference in the loadings on the aggregate volatility risk factor between high and low volatility firms should totally explain the idiosyncratic volatility effect and the stronger idiosyncratic volatility effect for growth firms. The aggregate volatility factor should also significantly contribute to explaining the value effect and why it is stronger for high volatility firms. In my empirical tests, leaning on Campbell (1993) and Chen (2002), I define the aggregate volatility factor as the zero-cost portfolio long in the firms with the lowest (most negative) return sensitivity to aggregate volatility increases and short in the firms with the highest (most positive) sensitivity.

I can also use Proposition 3 to test the hedging ability of high volatility, growth, and high volatility growth firms against adverse business-cycle shocks in a more conventional fashion. In the CAPM, lower risk premium means lower betas. Proposition 3 can be rephrased in terms of betas to show that in the conditional CAPM the betas of high volatility, growth, and high volatility growth firms are lower in recessions than in booms (details are available from the author). This hypothesis can be easily tested empirically.

Theoretically, the ICAPM is a more fruitful framework to explain the three idiosyncratic volatility effects than the conditional CAPM. The conditional CAPM assumes investors have no hedging demands and only care about the market risk. The idiosyncratic volatility hedging channel in the conditional CAPM is limited to the negative correlation between the market beta and the market risk premium, which produces negative unconditional CAPM alphas for high volatility, growth, and high volatility growth firms.

Beyond that, in the ICAPM the hedging channel also means that these three types of firms provide additional consumption when it is most needed to increase savings. The reasons to increase savings after volatility increases are worse future investment opportunities and lower future consumption (Campbell, 1993) and higher future volatility and the precautionary motive (Chen, 2002). Also, the ICAPM captures the hedge coming from the fact that the value of growth options increases with volatility.

As in the previous subsection, the results in this subsection can be reformulated using any option-like dimension of the firm. The implication is that no matter which option-like dimension of the firm (market-to-book, leverage, etc.) is creating the idiosyncratic volatility discount, it should be explained by lower sensitivity of high volatility firms to negative business-cycle news and their lower risk in recessions.

3 Data and Descriptive Statistics

3.1 Data Sources

My data span the period between July 1963 and December 2006. Following AHXZ, I measure idiosyncratic volatility as the standard deviation of the Fama-French (1993) model residuals, which is fitted to daily data. I estimate the model separately for each firm-month, and compute the residuals in the same month. I require at least 15 daily returns to estimate the model and idiosyncratic volatility. I sort firms into idiosyncratic volatility quintiles at the end of each month using NYSE breakpoints and compute the returns over the next month using monthly return data from CRSP. Firms are classified as NYSE if the `exchcd` listing indicator from the CRSP events file at the portfolio formation date is equal to 1.

I do not include in my analysis utilities (SIC codes 4900-4999) and financials (SIC codes 6000-6999). I also include only common stock (CRSP codes 10 and 11). I construct the book-to-market ratio using the Compustat data, where the market value is defined as above and the book value is book equity (Compustat item #60) plus deferred taxes (Compustat item #74). The book value of deferred taxes is set to zero for firms that do not report it. To compute the market-to-book, I use the current year book value for firms with the fiscal year end in June or earlier or the previous year book value for firms with later fiscal year end, to ensure that the book value is available before the date of portfolio formation.

I use monthly cum-dividend returns from CRSP and complement them by the delisting returns from the CRSP events file. Following Shumway (1997) and Shumway and Warther (1999), I set delisting returns to -30% for NYSE and AMEX firms (CRSP `exchcd` codes equal to 1, 2, 11, or 22) and to -55% for NASDAQ firms (CRSP `exchcd` codes equal to 3 or 33) if CRSP reports missing or zero delisting returns and delisting is for performance reasons. My results are robust to setting missing delisting returns to -100% or using no correction for the delisting bias.

I obtain the daily and monthly values of the three Fama-French factors and the risk-free rate from Kenneth French web site at http://mba.tuck.dartmouth.edu/pages/faculty /ken.french/.

To measure the return sensitivity to changes in aggregate volatility, I use daily changes in the old version of the VIX index calculated by CBOE and available from WRDS. Using the old version of VIX gives a longer coverage starting with January 1986. The VIX index measures the implied volatility of the at-the-money options on S&P100. For a detailed description of VIX, see Whaley (2000) and AHXZ.

I measure the return sensitivity to changes in the VIX index by running each firm-month the regression of the daily excess returns to the stock on the daily excess returns to the market and the VIX change in this day. I require at least 15 non-missing returns in a firm-month for the estimation. The BVIX factor is then defined as the value-weighted return differential between the most negative and most positive VIX sensitivity quintile. AHXZ use the FVIX factor instead, which is the factor-mimicking portfolio tracking the VIX index. I use a simpler procedure to form my aggregate volatility risk factor because of estimation error concerns.

To estimate the conditional CAPM, I employ four commonly used conditioning variables: the dividend yield, the default premium, the risk-free rate, and the term premium. I define the dividend yield, (DIV_t), as the sum of dividend payments to all CRSP stocks over the previous 12 months, divided by the current value of the CRSP value-weighted index. The default spread, (DEF_t), is the yield spread between Moody's Baa and Aaa corporate bonds. The risk-free rate is the one-month Treasury bill rate, (TB_t). The term spread, $(TERM_t)$, is the yield spread between ten-year and one-year Treasury bond. The data on the dividend yield and the risk-free rate are from CRSP. The data on the default spread and the term spread are from FRED database at the Federal Reserve Bank at St. Louis.

In the tests of my model against behavioral stories I use two measures of limits to arbitrage - residual institutional ownership, $RInst$, and the estimated probability to be on special, $Short$, which proxies for the severity of short sale constraints. I define institutional ownership of each stock as the sum of institutional holdings from Thompson Financial 13F database, divided by the shares outstanding from CRSP. If the stock is on CRSP, but not on Thompson Financial 13F database, it is assumed to have zero institutional ownership. Following Nagel (2004), I drop all stocks below the 20th NYSE/AMEX size percentile and measure residual institutional ownership for the remaining stocks as the residual from

$$log(\frac{Inst}{1-Inst}) = \gamma_0 + \gamma_1 \cdot log(Size) + \gamma_2 \cdot log^2(Size) + \epsilon \tag{19}$$

The estimated probability to be on special is defined as in D'Avolio (2002) and Ali and

Trombley (2006)

$$Short = \frac{e^y}{1+e^y}, \tag{20}$$

where

$$y = -0.46 \cdot log(Size) - 2.8 \cdot Inst + 1.59 \cdot Turn - 0.09 \cdot \frac{CF}{TA} + 0.86 \cdot IPO + 0.41 \cdot Glam \tag{21}$$

Equation (21) uses the coefficients estimated by D'Avolio (2002) for a short 18-month sample of short sale data. Ali and Trombley (2006) use the same formula to estimate the probability to be on special for the intersection of Compustat, CRSP, and Thompson Financial populations. They show that the estimated probability is closely tied to other short sale constraint measures in different periods.

In (21) $Size$ is defined as shares outstanding times the price per share and measured in millions, $Inst$ is institutional ownership, $Turn$ is turnover, defined as the trading volume over shares outstanding, CF is cash flow[5], TA are total assets (Compustat item #6), IPO is the dummy variable equal to 1 if the stock first appeared on CRSP 12 or less months ago, and $Glam$ is the dummy variable equal to 1 for three top market-to-book deciles.

3.2 Descriptive Statistics

In Table 1 I report descriptive statistics across the idiosyncratic volatility quintiles formed using the previous month idiosyncratic volatility and rebalanced each month. Panel A looks at the quintiles formed using the breakpoints for the whole CRSP population. In my sample, I confirm the findings of AHXZ that the idiosyncratic volatility discount is about 1% per month in value-weighted returns and even more in the Fama-French abnormal returns. In equal-weighted returns, though, it is only present in the Fama-French (1993) abnormal returns. The equal-weighted abnormal return differential between the lowest and the highest volatility quintile is estimated at 0.6% per month, t-statistic 2.99, versus the value-weighted abnormal return differential of 1.32% per month, t-statistic 6.86. The weaker idiosyncratic volatility discount in equal-weighted returns is not surprising, because the idiosyncratic volatility discount runs against the size effect, which is much stronger in equal-weighted returns.

[5]Following D'Avolio (2002) and Ali and Trombley (2006) I define cash flow as operating income before depreciation (Compustat item #178 plus Compustat item #14) less non-depreciation accruals, which are change in current assets (Compustat item #4) less change in current liabilities (Compustat item #5) plus change in short-term debt (Compustat item #34) less change in cash (Compustat item #1).

In the rest of the paper I will look at double sorts on idiosyncratic volatility and market-to-book. To keep all 25 portfolios balanced and non-negligible in terms of market cap percentage, I will use NYSE breakpoints to sort firms on both volatility and market-to-book. Therefore, in Panel B I look at idiosyncratic volatility quintiles that use NYSE breakpoints. Firms are classified as NYSE if the `exchcd` listing indicator from the CRSP events file is equal to 1. The `exchcd` indicator summarizes the listing history of the firm and reveals where the stock was listed at the portfolio formation date. It makes `exchcd` different from the `hexcd` listing indicator in the CRSP returns file, which reports the most recent listing. In Section 6.1 I show that using the `hexcd` indicator instead of `exchcd` creates a strong selection bias for the highest volatility firms. This bias contaminates the results in Bali and Cakici (2007) and explains why they find that the idiosyncratic volatility discount is not robust.

In Panel B the idiosyncratic volatility discount is smaller. It is absent in the raw returns, both equal-weighted and value-weighted, but is reliably present in the Fama-French alphas. The Fama-French alpha of the portfolio long in the lowest volatility quintile and short in the highest volatility quintile is 32 bp per month, t-statistic 2.22, for equal-weighted returns, and 59 bp per month, t-statistic 4.34, for value-weighted returns. It is twice smaller than what I get using CRSP breakpoints to form the quintiles, but still economically large and highly significant.

The fact that using NYSE breakpoints gives smaller values of the idiosyncratic volatility discount is not surprising. Both Panel A and Panel B show that the idiosyncratic volatility discount is driven primarily by the stocks in the highest volatility quintile. Because NYSE stocks are usually larger and less volatile, using NYSE breakpoints means pushing more stocks in the highest volatility quintile, and it depresses the idiosyncratic volatility discount.

In Panel C I estimate the Fama-French factor betas for each of the volatility quintiles (with NYSE breakpoints). I find that the market beta and the size beta strongly increase with volatility, and the HML beta strongly decreases with volatility, suggesting that the stocks in the highest volatility quintile are small and growth. It is confirmed in the last two rows of Panel C, which report size and market-to-book at the portfolio formation date. The highest volatility firms tend to be much smaller and have a much higher market-to-book that other firms.

4 Cross-Sectional Tests

4.1 Double Sorts

My model predicts that the idiosyncratic volatility discount increases with market-to-book and is absent for value firms. The prediction about the value effect is symmetric and implies that the value effect increases with idiosyncratic volatility. I first look at the 5-by-5 independent portfolio sorts on market-to-book and idiosyncratic volatility. The sorts are performed using NYSE (`exchcd`=1) breakpoints. The results are robust to using conditional sorting and/or CRSP breakpoints.

In Panel A of Table 2 I test these hypotheses for the Fama-French (1993) value-weighted abnormal returns. I use the formation month market capitalization for value-weighting. The Fama-French abnormal returns are defined as the alphas from separate time-series regressions fitted to each of the 25 portfolios. The results are robust to using raw returns or the CAPM alphas instead.

Panel A shows that the predictions of my model are strongly supported by the data. The magnitude of the idiosyncratic volatility discount monotonically increases with market-to-book from 10 bp per month (t-statistic 0.48) in the extreme value portfolio to 84 bp per month (t-statistic 3.92) in the extreme growth portfolio. The differenceis highly significant with t-statistic 2.80. In terms of statistical significance, the idiosyncratic volatility discount is confined to the three top market-to-book quintiles.

A similar pattern is observed for the value effect. It starts with the negative Fama-French alpha of -27.5 bp per month (t-statistic -1.50) in the lowest volatility quintile, monotonically increases across the idiosyncratic volatility quintiles, and ends up with the Fama-French alpha of 46 bp per month (t-statistic 2.47) for the highest volatility quintile. The highest idiosyncratic volatility quintile is the only one in which the Fama-French model cannot fully explain the value effect.

Equal-weighted alphas in Panel B give a similar picture. If the returns are equal-weighted, the idiosyncratic volatility discount increases from -32.5 bp, t-statistic -1.81 in the value quintile to 65 bp, t-statistic 3.34, in the growth quintile. The growth quintile is the only market-to-book quintile with the significant idiosyncratic volatility discount in equal-weighted returns. The difference in the idiosyncratic volatility discount between value firms and growth firms is highly significant with t-statistic 4.92.

The unexplained part of the value effect also increases with idiosyncratic volatility from 23 bp, t-statistic 1.86, in the lowest volatility quintile to the astonishing 1.2%, t-statistic 8.23, in the highest volatility quintile. The Fama-French model cannot explain the value effect in equal-weighted returns in the top three idiosyncratic volatility quintiles.

The portfolio that seems to generate the majority of these effects and represents the worst failure of the Fama-French model in the double sorts on market-to-book and volatility, is the highest volatility growth portfolio. In Panel A this portfolio witnesses the negative alpha of -56.5 bp, t-statistic -3.89, and earns the average raw return of 43 bp per month, very close to the average value of the risk-free rate in my sample. Such a low return to high volatility growth firms is totally consistent with my model. The model predicts that the firms with the highest volatility and the highest market-to-book should be the best hedges against aggregate volatility increases and should therefore earn the lowest expected return.

The bottom line of Table 2 is that the Fama-French model fails to explain the idiosyncratic volatility discount if market-to-book is high and it fails to explain the value effect if idiosyncratic volatility is high. The Fama-French model also fails to explain why the idiosyncratic volatility discount increases with market-to-book and why the value effect increases with idiosyncratic volatility. It proves the importance of the interaction between idiosyncratic volatility and growth options analyzed in my model.

4.2 Firm-Level Fama-MacBeth Regressions

To corroborate my findings in Table 2, I run firm-level Fama-MacBeth (1973) regressions in Table 3. In each month I regress the return to each firm on its market beta estimated using daily returns in the current month, and firm characteristics measured in the previous year. I use the percentage ranking of size, market-to-book and idiosyncratic volatility as the independent variables, because the untransformed variables are extremely skewed.

In the first three columns of Table 3 I make sure that the patterns already documented in the literature exist in my sample as well. I document the strong and significant size effect, value effect, and idiosyncratic volatility discount. I also show in the third column that the value effect is larger for high volatility firms.

In the fourth column I test the main prediction of my model by estimating the regression from Hypothesis 1. I regress returns on beta, size, market-to-book, idiosyncratic volatility, and the product of market-to-book and volatility. My model predicts that the coefficient of the product of market-to-book and volatility will be negative and highly significant. After I add the product, the sign of the coefficient on idiosyncratic volatility should change.

Table 3 shows that this prediction is strongly supported by the data. The product of market-to-book and volatility is extremely significant with t-statistic -6.31, and the idiosyncratic volatility has positive and insignificant coefficient. It means that the idiosyncratic volatility discount is absent for extreme value firms and is significantly increasing in market-to-book, which confirms my findings in Table 2.

The coefficient on market-to-book also changes the sign in the presence of its product with idiosyncratic volatility. It means that the interaction between growth options and idiosyncratic volatility predicted by my model can be strong enough to subsume the return effects usually attributed to either market-to-book or idiosyncratic volatility.

The magnitude of the coefficient on the interaction term suggests that its economic impact is large. From the estimates in the fourth column I predict that the idiosyncratic volatility discount will be $-(0.0016 \cdot (90-10) - 0.00024 \cdot (90-10) \cdot 10) = 0.07\%$ per month for extreme value firms (10% market-to-book percentile) and $-(0.0016 \cdot (90-10) - 0.00024 \cdot (90-10) \cdot 90) = 1.60\%$ per month for extreme growth firms (90% market-to-book percentile). The estimated strength of the idiosyncratic volatility discount for growth firms and its difference from the idiosyncratic volatility discount for value firms are larger than what I estimate in Table 2, because I use NYSE breakpoints there. When I use CRSP breakpoints for the double sorts (results not reported), I estimate the idiosyncratic volatility discount, defined as the value-weighted Fama-French alpha, for the growth quintile at 1.73%, 1.46% difference from the value quintile, which is very close to the estimates from the cross-sectional regression.

Making further use of Hypothesis 1, I take the ratio of the coefficients on idiosyncratic volatility and its product with market-to-book to measure the percentage of firms with no growth options. The result implies that 6.6% of the firms in my sample have no growth options, which is quite plausible.

Ali et al. (2003) run a similar regression in their Table 3 and find the right and significant sign on the product of volatility and market-to-book. They fail to find that adding the product on the left-hand side flips the signs of market-to-book and idiosyncratic volatility. The main difference between our research designs is that they use size-adjusted returns on the left-hand side of the regression. Keeping in mind the negative relation between the size effect and the idiosyncratic volatility discount, I suspect that the crude size adjustment they use can overstate the idiosyncratic volatility discount.

Controlling for idiosyncratic volatility increases the magnitude and significance of the size effect. The slope of the size variable increases by 50% and the t-statistic nearly doubles when I compare the first column of Table 3 with any other column. This result is driven by the negative correlation between size and idiosyncratic volatility. I conclude that the size effect is likely to be stronger than previously thought. It may seem insignificant in recent years (see Schwert, 2003) just because it runs counter to the idiosyncratic volatility discount and both idiosyncratic volatility and its effect on returns have also recently increased (Campbell et al., 2001, and AHXZ, Table XI).

In Barinov (2007b) I explore the link between the size effect and the idiosyncratic volatility discount further and find that the change in the slope is no coincidence. Barinov (2007b) shows that if one sorts on the size variable orthogonalized to idiosyncratic volatility, the size effect in returns doubles and restores its significance in all time periods.

5 Time-Series Tests

5.1 Is Aggregate Volatility Risk Priced?

Changes in aggregate volatility provide information about future investment opportunities and future consumption. In Campbell (1993), an increase in aggregate volatility implies that in the next period risks will be higher and consumption will be lower. Consumers, who wish to smoothen consumption, have to save and cut current consumption if aggregate volatility goes up. Chen (2002) also notes that higher current aggregate volatility means higher aggregate volatility in the future. Therefore, consumers will build up precautionary savings and cut current consumption in response to volatility increases. Both Campbell (1993) and Chen (2002) predict that the most negatively correlated with changes in aggregate volatility stocks earn a risk premium. These stocks are risky because their value drops when consumption has to be cut to increase savings.

Ang, Hodrick, Xing, and Zhang (2006) (henceforth - AHXZ) show that stocks with positive return sensitivity to the innovations in the VIX index indeed earn about 1% per month less than stocks with negative sensitivity. The VIX index measures the implied volatility of the S&P100 options and behaves like a random walk. The change in VIX is therefore a good proxy for the innovation in expected aggregate volatility.

In this subsection I extend the findings of AHXZ by showing that aggregate volatility risk is priced for several portfolio sets and that the ICAPM with the aggregate volatility risk factor performs at least as well as the Fama-French model.

I measure return sensitivity to the aggregate volatility movements by regressing firm daily excess returns on excess market returns and the change in the VIX index, as AHXZ do. I run these regressions separately for each firm-month, and require at least 15 non-missing observations for each firm-month. The sample period is from February 1986 to December 2006, because the CBOE data on VIX start in January 1986, and I lag the return sensitivities to VIX changes by one month to form the BVIX factor.

In each month, I sort firms by their VIX sensitivity in the previous month and form the BVIX factor portfolio. It is long in the lowest sensitivity quintile and short in the highest sensitivity quintile. The BVIX factor is the factor I use to augment the CAPM to explain the three idiosyncratic volatility effects. AHXZ employ a more sophisticated procedure of forming the factor-mimicking portfolio (FVIX), which tracks the changes in VIX. I choose a simpler procedure to form the BVIX factor mainly because of estimation error concerns.

In Panel A of Table 4 I verify that sorting stocks on return sensitivity to the VIX changes creates a spread in returns unrelated to other priced factors. Panel A shows that the return differential is between 86 to 98 bp per month, depending on the risk factors I control for. It is slightly lower than what AHXZ (2006) document in 1986-2000.

In Panel B of Table 4, I use the Gibbons, Ross, and Shanken (1989) (hereafter - GRS) test statistic to compare the performance of the CAPM, the Fama-French model, and the ICAPM with the BVIX factor. The GRS statistic tests whether the alphas of all portfolios in a portfolio set are jointly equal to zero, and whether the BVIX betas of all portfolios are jointly equal to zero. The GRS statistic gives more weight to more precise alpha estimates, which usually come from low volatility stocks. Because BVIX should explain the alphas of high volatility firms, the GRS statistic estimates the usefulness of BVIX quite conservatively. The tests in Panel B use equal-weighted returns to the portfolios sets. Using value-weighted returns instead does not change the conclusions.

Panel B brings me to three main conclusions. First, the BVIX betas are highly jointly significant for all portfolio sets. Second, adding the BVIX factor to the CAPM always significantly improves the GRS statistic for alphas, though it still remains significant. The improvement of the GRS statistic is about 17% for the 25 idiosyncratic volatility - market-to-book portfolios and about 4% and 6% for the 25 size - market-to-book portfolios and the 48 industry portfolios. Third, for the 25 idiosyncratic volatility - market-to-book portfolios and the 48 industry portfolios the ICAPM with BVIX performs better in terms of alphas than the Fama-French model.

In Barinov (2007a) I take a more detailed study of the BVIX factor pricing ability. I find that the improvement over the CAPM in the 25 size - market-to-book portfolios comes from the BVIX factor ability to explain the abnormally low returns to the smallest growth firms and, consequentially, the puzzling negative size effect in the extreme growth quintile. The respective alphas are reduced by more than a half and become insignificant. I also show that the BVIX factor provides an explanation of the new issues puzzle. The ICAPM with BVIX reduces the alphas of the IPO and SEO portfolios by about 45% and makes them insignificant. The BVIX factor is also successful in explaining the abysmal performance of new issues performed by small firms and growth firms, and its difference with the performance of the new issues performed by large firms and value firms. Coupled with the evidence in Table 5, it suggests that the BVIX factor is not an *ad hoc* patch on the CAPM. Rather, it is a priced factor, helpful in resolving many puzzles the existing asset-pricing models cannot fully address and significant in explaining returns to a wide variety of portfolios.

5.2 The Three Idiosyncratic Volatility Effects and Aggregate Volatility Risk

My model establishes an economy-wide idiosyncratic volatility hedging channel. As the economy slides into recession and expected aggregate volatility increases, the corresponding increase in idiosyncratic volatility mutes the effect of the bad news on growth options for two reasons. First, higher idiosyncratic volatility in my model means lower risk of growth options. Hence, the presence of idiosyncratic volatility makes smaller any increase in the risk premium of growth options caused by the recession, and also makes smaller any corresponding drop in their value. Second, higher idiosyncratic volatility makes growth options more valuable, which also mutes any drop in their value the recession may cause.

In Propositions 3 and 4, I show that both mechanisms behind the idiosyncratic volatility channel are stronger for high volatility, growth, and especially high volatility growth firms. It makes these three types of firms good hedges against aggregate volatility risk, as their returns covary least negatively with the changes in aggregate volatility. Therefore, aggregate volatility risk should explain the three idiosyncratic volatility effects: the idiosyncratic volatility discount, the stronger value effect for high volatility firms, and the stronger idiosyncratic volatility discount for growth firms.

To get the idea of how helpful the BVIX factor may be in explaining the idiosyncratic volatility effect, I report its betas from the ICAPM with BVIX run at monthly frequency for each of the 25 idiosyncratic volatility - market-to-book portfolios. A negative BVIX beta implies that the portfolio returns are positive when the stock with the most negative correlation with aggregate volatility lose value. Hence, portfolios with negative BVIX betas are hedges against aggregate volatility risk.

Table 5 shows that the BVIX betas are closely aligned with the Fama-French alphas in Table 2. High idiosyncratic volatility firms have negative BVIX betas that are significantly lower than the BVIX betas of low volatility firms. In all market-to-book quintiles the BVIX betas decrease almost monotonically as idiosyncratic volatility increases. Growth firms also have significantly lower BVIX betas than value firms. With few exceptions, in all volatility quintiles the BVIX betas decrease monotonically with market-to-book. It suggests that both the idiosyncratic volatility discount and the value premium can be at least partly explained by sensitivity to aggregate volatility.

Most importantly, the BVIX betas spread between high and low volatility firms increases with market-to-book, and the BVIX betas spread between value and growth firms increases in idiosyncratic volatility. Panel A of Table 4 estimates the factor premium earned by BVIX at 0.9% per month, which makes the spread in the BVIX betas enough to explain from 50% to 75% of the idiosyncratic volatility discount. In Panel A of Table 5 that uses value-weighted returns the BVIX betas of low and high volatility firms differ by only 0.199 (t-statistic 1.99) in the value quintile. The differential increases to 0.604 (t-statistic 3.57) in the growth quintile. The difference is highly significant with t-statistic 3.37. Similarly, the BVIX betas spread between value and growth is zero in the bottom three idiosyncratic volatility quintiles, but it increases to 0.260 (t-statistic 2.32) and 0.382 (t-statistic 2.75) in the fourth and the fifth volatility quintile. I also observe a highly negative BVIX beta of -0.395, t-statistic -2.94, for the highest volatility growth portfolio, which shows it is a very good hedge against aggregate volatility increases.

The results in Panel B that looks at equal-weighted returns are, if anything, stronger. The BVIX betas spread between low and high volatility in the growth quintile increases to 0.702, t-statistic 2.67, and is 0.526, t-statistic 3.57, higher than the similar spread in the value quintile. The BVIX betas spread between value and growth is also slightly higher at 0.412, t-statistic 4.01, and the BVIX beta of the highest volatility growth portfolio becomes as low as -0.527, t-statistic -2.56.

The conclusion from Table 5 is that, consistent with my model, high volatility firms, growth firms, and especially high volatility growth firms hedge against aggregate volatility risk. Their prices tend to go up when the prices of the firms with the most negative correlation with aggregate volatility go down. Their BVIX betas are significantly lower than the BVIX betas of low volatility, value and low volatility value firms, which can explain a large fraction of the three idiosyncratic volatility effects.

5.3 The Three Idiosyncratic Volatility Effects, the Conditional CAPM, and the Business Cycle

The return sensitivity to changes in the BVIX index is not the most common measure of firm exposure to economy-wide shocks. In this section, I use a more popular framework of the conditional CAPM to corroborate the findings from the previous subsection. In my tests I rely on Proposition 3 that predicts that the risk exposure of high idiosyncratic volatility, growth, and especially high volatility growth firms tends to decrease in recessions, when risk is higher.

I use four arbitrage portfolios that measure the three idiosyncratic volatility effects. The IVol portfolio captures the idiosyncratic volatility discount. It goes long in low volatility firms and short in high volatility firms. The IVolh portfolio does the same for growth firms only to capture the stronger idiosyncratic volatility discount in the growth quintile. The HMLh (HMLl) portfolios look at the value effect for high (low) volatility firms. The HMLl portfolio is not particularly challenging for the unconditional models and is used only for comparison with HMLh. The IVol55 portfolio is long in the highest volatility growth firms and short in the one-month Treasury bill.

Proposition 3 implies that when the expected market risk premium is high, the market beta of the first four (last) portfolios is higher (lower) than in good states of the world. In what follows, I assume that the expected market risk premium and the conditional beta are linear functions of the four commonly used business cycle variables - dividend yield, default spread, one month Treasury bill rate, and term spread. I define the bad state of the world, or recession, as the months when the expected market risk premium is higher than its in-sample mean.

In Table 6, I look at the average market betas across the states of the world for the five arbitrage portfolios I study. The expected market return is estimated as the fitted part of the regression

$$MKT_t = \gamma_0 + \gamma_1 \cdot DIV_{t-1} + \gamma_2 \cdot DEF_{t-1} + \gamma_3 \cdot TB_{t-1} + \gamma_4 \cdot TERM_{t-1} + \epsilon_t \tag{22}$$

To estimate the conditional CAPM beta, I run the regression

$$Ret_{it} = \alpha_i + (\beta_{0i} + \beta_{1i} \cdot DIV_{t-1} + \beta_{2i} \cdot DEF_{t-1} + \beta_{3i} \cdot TB_{t-1} + \beta_{4i} \cdot TERM_{t-1}) \cdot MKT_t + \epsilon_{it} \tag{23}$$

and define the conditional beta as

$$\beta_i = \beta_{0i} + \beta_{1i} \cdot DIV_{t-1} + \beta_{2i} \cdot DEF_{t-1} + \beta_{3i} \cdot TB_{t-1} + \beta_{4i} \cdot TERM_{t-1} \tag{24}$$

The left part of Table 6 looks at value-weighted returns and shows very strong evidence in favor of my model. For value-weighted returns I find that for the IVol and IVolh the conditional CAPM betas are by 0.202 and 0.288 higher in recessions than in expansions (standard errors 0.051 and 0.061, respectively). It means that exploiting the idiosyncratic volatility discount implies high exposure to business cycle risk. Also, the IVol55 portfolio turns out to be a good hedge against adverse business cycle movements, as its beta is by 0.177 (standard error 0.035) lower in recessions than in expansions. The right part of Table 6, which uses equal-weighted returns, shows very similar results.

I also find, consistent with Proposition 3, that the betas of the HMLl portfolio do not show reliable dependence on business cycle. The HMLl beta differential between recessions and expansions is small and its sign depends on whether I use value-weighted or equal-weighted returns. On the other hand, the CAPM beta of HMLh portfolio is by 0.286 higher in recessions (standard error 0.035) for value-weighted returns and by 0.223 higher in recessions (standard error 0.036) for equal-weighted returns. The difference in the conditional beta sensitivity to business cycle between HMLl and HMLh reinforces the conclusion from Table 3 and Table 5 that the value effect is at least partly driven by the interaction of growth options and volatility.

Petkova and Zhang (2005) perform a similar analysis for the HML portfolio. In a time frame very similar to mine they find that the conditional CAPM beta of the HML portfolio is only by 0.05 higher in recessions than in expansions, which is two to six times smaller than the spread in conditional betas I find for my portfolios.

A natural question to ask is whether the variation in the conditional betas is likely to be enough to explain the idiosyncratic volatility effects. In unreported results, I find that the expected and realized market risk premium is higher in recessions by about 1% per month. Coupled with the variation in betas, for example, of the IVol portfolio, the conditional CAPM is likely to explain only 20 bp of the 60 bp idiosyncratic volatility discount, i.e. only a third of the anomaly. I hypothesize, and show in the next subsection, that the failure of the conditional CAPM occurs because it ignores the hedging demands captured by the ICAPM.

5.4 Explaining the Three Idiosyncratic Volatility Effects

In Table 7, I test the ability of a variety of asset-pricing models to explain the idiosyncratic volatility effects represented by the returns to the four portfolios - IVol, IVolh, HMLh, and IVol55 - described at the start of the previous subsection. The sample period is determined by the availability of the VIX index and goes from February 1986 to December 2006.

In the first two columns I present the alphas from the unconditional CAPM and the unconditional Fama-French model. The unconditional CAPM turns out to be incapable of explaining either the value-weighted or the equal-weighted returns to any of the portfolios except for the equal-weighted IVol portfolio, which has the insignificant equal-weighted alpha. The magnitude of the CAPM alphas is about 1% per month. The Fama-French model can handle the equal-weighted IVolh portfolio and the value-weighted IVol55 in addition to the equal-weighted IVol portfolio, but desperately fails on the rest. The significant Fama-French alphas are around 0.6% per month.

In the next pair of columns, I estimate the ICAPM with the BVIX factor and find that it perfectly explains the returns to all portfolios except for HMLh. For example, the IVolh portfolio measures the idiosyncratic volatility discount in the extreme growth quintile and possesses the value-weighted CAPM and Fama-French alphas of 114 bp per month (t-statistic 3.03) and 67 bp per month (t-statistic 2.08), respectively. Adding the BVIX factor to the CAPM cuts the alpha to 54 bp per month, t-statistic 1.38.

The explanatory power of BVIX is also visible in the fourth column, which reports the BVIX betas of the four portfolios. The BVIX betas vary from 0.4 to 0.7 and are highly statistically significant. Trying to exploit the three idiosyncratic volatility effects exposes the investor to extremely high levels of aggregate volatility risk. The returns to the IVol, IVolh, and HMLh portfolios tend to be very low when aggregate volatility is high and consumption is low.

The fact that the BVIX does not completely explain the returns to the HMLh portfolio is not surprising if one thinks there is something to the value effect except for the interaction between growth options and idiosyncratic volatility. What my model predicts is that BVIX should be useful in explaining the returns to HMLh, and the highly significant BVIX betas of HMLh portfolio suggest it is useful. The reduction in the HMLh alpha brought about by adding BVIX to the CAPM is 40 bp, less than what the Fama-French model is able to achieve, but still quite sizeable.

As I argued in Section 2.2, there are several reasons why BVIX, the aggregate volatility risk factor, should explain the three idiosyncratic volatility effects. In the conditional CAPM framework, which assumes that investors do not care about intertemporal substitution, the BVIX factor should be regarded as a proxy for what Jagannathan and Wang (1996) call the beta instability risk. Essentially, the conditional CAPM says that the BVIX factor is a quasi-factor that should eliminate the negative bias in the unconditional alphas, created by the negative correlation between the market beta and the market risk premium. In the conditional CAPM, the BVIX factor can capture only a part of the beta instability risk, because there are other conditioning variables, beyond aggregate volatility, that are related both to idiosyncratic volatility and expected market risk premium.

In the ICAPM, the idiosyncratic volatility hedging channel has more impact, which includes the role it plays in the conditional CAPM. The ICAPM embraces the beta instability risk. It also points out that the smaller increase of risk premium in recession means the smaller decrease in current stock prices as the news about the recession arrive. The smaller decrease in price yields additional consumption when it is most valuable because both the future investment opportunities become worse (Campbell, 1993) and the higher future volatility calls for more precautionary savings (Chen, 2002).

Also, growth options hedge against the above negative consequences of aggregate volatility because their value increases as volatility increases. This effect is naturally stronger for growth firms and high volatility firms. This is another reason why growth firms and high volatility firms provide additional consumption when it is most needed and why they are good hedges against aggregate volatility risk.

In sum, the explanatory power of the conditional CAPM and the ICAPM should overlap, but none of them should subsume the other. The BVIX factor can be an imperfect proxy for the beta instability risk, but it is also capable of capturing other dimensions of risk that are absent in the conditional CAPM. The ability to capture other dimensions of risk is the reason why I expect the ICAPM with BVIX to outperform the conditional CAPM in explaining the three idiosyncratic volatility effects.

The fifth column of Table 7 reports the alphas of the conditional CAPM in Section 5.3, equation (23). I assume that the conditional market beta is a linear function of the dividend yield, the default spread, the one-month Treasury bill rate, and the term spread. The alphas are significantly smaller than the unconditional CAPM alphas in the first column, usually by about 25%, or by 20-35 bp, quite close to what the back-of-envelope calculation in the end of Section 5.3 predicts. However, the conditional CAPM alphas are generally significant and considerably lag behind the Fama-French and ICAPM alphas.

In columns six and seven I look at the conditional ICAPM, where only the market beta is conditioned on the four business cycle variables. The point of this exercise is to evaluate the overlap between the BVIX factor and the conditioning variables. As I argued above, I expect some overlap, and some unique risk in both the BVIX factor and the conditioning variables, with BVIX picking up more significant risks.

In column six I look at the alphas and see that they are uniformly reduced by 10-15 bp compared to the ICAPM alphas in the third column. Most of them are reduced from insignificant values and the significant ones do not become insignificant. The results suggest that the BVIX factor captures the majority of the beta instability risk, but some of it is still captured by the conditioning variables only. The overlap between the conditioning variables and BVIX is further confirmed by column seven, which reports the BVIX betas in the conditional ICAPM. The BVIX betas uniformly decrease by about 0.1 and even become marginally significant in equal-weighted returns. However, most of the betas remain large and statistically significant, which confirms that there are important risks captured by the BVIX factor, but not by the conditioning variables.

6 Alternative Explanations

6.1 Is the Idiosyncratic Volatility Discount Robust? Revisiting Bali and Cakici (2007).

In a recent paper, Bali and Cakici (2007) claim that the idiosyncratic volatility discount is not robust to reasonable changes in the research design. In particular, they argue that measuring idiosyncratic volatility from monthly data or looking at NYSE only firms eliminates the idiosyncratic volatility discount.

When I try to mimic the results in Bali and Cakici (2007), I find that they are contaminated by selection bias. When Bali and Cakici look at NYSE only firms, they define a NYSE firm using the current listing reported in the `hexcd` listing indicator from the CRSP returns file. It creates a strong selection bias, because only good performers remain NYSE firms from the portfolio formation date till now. Bad performers, even if they were NYSE firms at the portfolio formation date, are likely to be subsequently downgraded to NASDAQ or even OTC, and therefore they do not make it into the Bali and Cakici "NYSE only" sample. On the other hand, good performers, even if they were NASDAQ at the portfolio formation date, are likely to make it into the "NYSE only" sample, because they may be upgraded to NYSE since then. This selection bias is evidently stronger for high idiosyncratic volatility firms, which are more likely to be upgraded or downgraded.

The natural way to avoid the selection bias is to look at the historical listing recorded in the `exchcd` indicator from the CRSP events file and use its value at the portfolio formation date to classify firms as NYSE firms. When I do it, I find that the idiosyncratic volatility discount in the NYSE only sample is actually larger than in the whole CRSP population.

I follow Bali and Cakici (2007) in measuring idiosyncratic volatility from monthly data. I define it as the standard deviation of the Fama-French model residuals, where the Fama-French model is fitted to monthly returns from 24 to 60 months ago (at least 24 valid observations are required for estimation). The monthly idiosyncratic volatility portfolios are rebalanced at the end of each month and held for one month afterwards. The daily idiosyncratic volatility measure in Bali and Cakici (2007) is the same as the one I use throughout the paper.

In Table 8 I look at the idiosyncratic volatility discount in the NYSE only sample. Panel A shows equal-weighted returns to the portfolios formed using the volatility from daily data, and Panel B shows equal-weighted returns to the portfolios formed using the volatility from monthly data. In the first two rows, I mimic Bali and Cakici (2007) by using `hexcd` from the CRSP returns file to classify firms as NYSE.

The raw returns are within 1 bp per month of what Bali and Cakici (2007) show in Table 2, Panel B, and in Table 4, Panel B. It convinces me that they were using the `hexcd` listing indicator, even though they are not explicit about it. In raw equal-weighted returns the idiosyncratic volatility discount turns into the idiosyncratic volatility premium of 25 bp (t-statistic 1.08) in Panel A and 51 bp (t-statistic 1.87) in Panel B. The respective Fama-French alphas show a small idiosyncratic volatility discount of 32 bp (t-statistic 2.67) and 3 bp (t-statistic 0.27).

When I matched Bali and Cakici (2007) in the top row of Table 8, I ignored delisting returns as they apparently did. Adding the delisting returns back increases the idiosyncratic volatility discount by 3 bp per month, as shown in the third row.

In the fourth row, I use the value of the `exchcd` listing indicator from the CRSP events file at the portfolio formation date to classify firms as NYSE. The effect of removing the selection bias created by using `hexcd` is enormous - the alphas of the highest volatility quintiles go down by 55 bp per month, and the idiosyncratic volatility discount jumps up by the same amount. In the true NYSE only sample it is even higher that in the CRSP population at 85 bp per month, t-statistic 6.30, for the sorts on the daily volatility measure, and at 67 bp per month, t-statistic 4.87, for the sorts on the monthly measure.

Overall, Table 8 demonstrates that Bali and Cakici (2007) fail to find the idiosyncratic volatility discount because of the pitfalls in their research design. Once I eliminate the selection bias that contaminate their results, I find the idiosyncratic volatility discount alive and well exactly for the cases where they claimed to find the greatest evidence against it.

6.2 Behavioral Stories

Several recent empirical papers find evidence consistent with the behavioral explanation of the idiosyncratic volatility discount. The behavioral story is based on the Miller (1977) argument that under short sale constraints firms with greater divergence of opinion about their value will be more overpriced as a result of the winner's curse. Consistent with this idea, Nagel (2004) and Boehme, Danielsen, Kumar, and Sorescu (2006) find that the idiosyncratic volatility discount is much stronger for the firms with low institutional ownership and high short interest.

Gompers and Metrick (2001) find that institutional ownership is negatively related to market-to-book, and D'Avolio (2002) estimates that the costs of shorting are higher for growth stocks. It means that limits to arbitrage are likely to be higher for growth stocks, and the relation between them and the idiosyncratic volatility discount can therefore drive my cross-sectional results.

A somewhat different behavioral story, yet unexplored in the literature, is that in my setup market-to-book can be just another measure of overpricing, and my cross-sectional results arise only because double sorts always create more variation in returns than single sorts. If it is the case, I would expect the interaction of volatility and market-to-book to be stronger for the stocks with higher limits to arbitrage.

Due to availability of the institutional ownership data, the sample period in this subsection starts in April 1980. I also restrict my sample to the stocks above the 20th NYSE/AMEX size percentile, since the other stocks usually have zero institutional ownership. I follow Nagel (2004) in looking at residual institutional ownership, which is orthogonalized to size (see equation (19)).

I do not have access to the short interest data and use the estimated probability that the stock is on special. The exact formula is given in (20) and (21) (see Section 3.1). It uses the coefficients estimated by D'Avolio (2002) for a short 18-month sample of short sale data. Ali and Trombley (2006) use the same formula to estimate the probability to be on special for the intersection of Compustat, CRSP, and Thompson Financial populations. They show that it is closely tied to other short sale constraint measures in different periods.

Panel A of Table 9 looks at the interaction between the idiosyncratic volatility discount and the residual institutional ownership in the firm-level cross-sectional regressions. In the first three columns I verify that the previously documented phenomena are present in my sample. I start with regressing returns on the current month beta, size, market-to-book, and idiosyncratic volatility for firms with non-missing residual institutional ownership only. Then I add the residual institutional ownership and its product with idiosyncratic volatility. I subtract 100 from the percentage ranking of institutional ownership so that the slope of the idiosyncratic volatility in the presence of its product with institutional ownership measured the idiosyncratic volatility discount for the highest, not lowest, institutional ownership firms. All independent variables are percentage ranks, and the sample period is from April 1980 to December 2006.

In the first column I find that in my sample the idiosyncratic volatility discount is large and significant with t-statistic 3.93. The second column of Panel A I confirm the return premium earned by the stocks with the highest residual institutional ownership (see Chen, Hong, and Stein, 2002, and Nagel, 2004). Controlling for the premium does not reduce the estimated magnitude of the idiosyncratic volatility discount. In the third column I find that the product of idiosyncratic volatility and residual institutional ownership is significantly positive with the t-statistic of 5.77 and the idiosyncratic volatility discount becomes insignificant in its presence. It confirms another result in Nagel (2004) that the idiosyncratic volatility discount is limited to the lowest institutional ownership stocks.

In the fourth column of Panel A I show that the dependence of the idiosyncratic volatility discount on institutional ownership is distinct from the effect in Nagel (2004). I include in the regression both the product of volatility and market-to-book and the product of volatility and residual institutional ownership. Both products are highly significant and their magnitude diminishes only slightly compared to the cases when they are used alone. I conclude that while there is supporting evidence for the behavioral story, the effect predicted by my model is clearly distinct from it.

In the fifth column I add the product of idiosyncratic volatility, market-to-book, and institutional ownership. The alternative behavioral story I develop in this subsection implies that it should have a positive sign and drive away the product of idiosyncratic volatility and market-to-book. I find that the interaction of volatility and market-to-book matters significantly more for low institutional ownership firms, consistent with the behavioral story. The interaction of volatility and market-to-book for the highest institutional ownership stocks is only a half of its value in column four and its t-statistic is -1.87. However, for the firms with institutional ownership above the 10th percentile the product of idiosyncratic volatility and market-to-book restores significance.

In Panel B I repeat the analysis for the probability to be on special, which is my proxy for short sale constraints. In the first column I find that the idiosyncratic volatility discount is alive and well in the sample with non-missing probability to be on special. In the second column I confirm the results of Asquith, Patak, and Ritter (2005) that higher short sale costs imply lower future returns, as the Miller (1977) model would imply. In the third column of Panel B I find that the product between idiosyncratic volatility and the probability to be on special is negative and significant with t-statistic -4.51. It means that the strength of the idiosyncratic volatility discount strongly depends on the level of short sale constraints, as Boehme et al. (2006) show. I also find that the idiosyncratic volatility discount is zero for the lowest short sale constraints firms.

In column four of Panel B I find that the interaction of volatility and market-to-book predicted by my model is distinct from the effect in Boehme et al. (2006). The product of idiosyncratic volatility and market-to-book does not change its magnitude and significance in the presence of the product of idiosyncratic volatility and the probability to be on special.

In the last column of Panel B I find that the product of volatility, market-to-book, and the probability to be on special makes the two other products insignificant, and it is insignificant itself. It suggests that this product should not probably be there and it only spoils the model because it is correlated with both the product of volatility and market-to-book and the product of volatility and the probability to be on special.

A possible problem with the regression in the last column is that the probability to be on special includes the glamour dummy and is correlated with market-to-book by construction. In untabulated results I try calculating the probability to be on special without the glamour dummy. I find that the regression in the last column gives the same result, with the significance of the volatility and market-to-book product slightly increased.

Overall, the results in Table 9 are inconclusive. While the results in my paper clearly are not subsumed by the Nagel (2004) or Boehme et al. (2006) stories, the interaction of volatility and market-to-book does not explain the evidence in Nagel (2004) and Boehme et al. (2006) either. Moreover, its strength depends on institutional ownership. To understand better the nature of what I label "behavioral effects" in the cross-sectional regressions, I run covariance-based tests in Table 10.

In Table 10 I measure the idiosyncratic volatility discount in each limits-to-arbitrage quintile using the value-weighted abnormal returns from three asset-pricing models - the CAPM, the Fama-French model, and the ICAPM with BVIX. The behavioral story behind the idiosyncratic volatility discount and Table 9 suggest that the idiosyncratic volatility discount should be stronger for high limits-to-arbitrage firms. My model, on the other hand, predicts that all cross-sectional variation in the idiosyncratic volatility discount should be related to risk.

Both Panel A (residual institutional ownership) and Panel B (probability to be on special) reach the same conclusion. Even the CAPM alphas do not confirm that the idiosyncratic volatility discount is reliably different between the quintiles with the lowest and the highest limits to arbitrage. While the difference in the CAPM alphas is large (about 80 and 70 bp), it is significant only at the 10% level. Using the Fama-French alphas instead reduces the difference to about 30 bp and 10 bp with minuscule t-statistics. Also, both the Fama-French model and the ICAPM can explain the idiosyncratic volatility discount in all limits-to-arbitrage quintiles, except, probably, the lowest institutional ownership quintile.

The fact that the BVIX betas are flat across the limits-to-arbitrage quintiles is consistent with Table 9. It means, as shown by columns four in Table 9, that the behavioral effects and the effects predicted by my model are unrelated.

Overall, Table 10 questions the reliability of the behavioral effects. The difference between Table 9 and Table 10 can come from two sources. First, it is possible that firm characteristics are poor risk controls. Second, it is possible that the behavioral effects in the firm-level regressions are caused by some outliers, which are muted in portfolios. Whichever of the two is true, the absence of the behavioral effects in the portfolios alphas seems to imply that they are not real.

Nagel (2004) also performs five-by-five sorts on idiosyncratic volatility and institutional ownership. He finds that the idiosyncratic volatility discount, measured as the Fama-French alpha, is reliably different for low and high institutional ownership firms. Nagel (2004) uses the data from September 1980 to September 2003, while I use the data from February 1986 to December 2006. In unreported analysis I find the difference in our results is driven by the period from September 1980 to January 1986. That is, the idiosyncratic volatility discount depends on institutional ownership only in the early 1980s.

Boehme et al. (2006) do find that the idiosyncratic volatility discount depends on the short sale constraint in the portfolio sorts. However, their research designs is too different from mine to allow simple comparison. First, they use direct data on short interest from NYSE and NASDAQ to sort on short sale constraints. Second, they augment the Fama-French model with the momentum factor to compute the alphas. Third, they perform four-by-twenty sorts. Without the data on short interest it is difficult to assess what drives the difference in our results.

7 Conclusion

My paper presents a real options model, which explains the idiosyncratic volatility discount and the stronger value effect for high volatility firms. The basic intuition driving these two results is that high idiosyncratic volatility decreases the risk of growth options by making them less sensitive to the changes in the underlying asset value. Because in my model the pricing effects of idiosyncratic volatility work through growth options, it generates a new empirical prediction that the idiosyncratic volatility discount is stronger for growth firms and absent for value firms.

My model predicts that aggregate volatility risk explains the idiosyncratic volatility discount. Aggregate volatility risk should also explain why idiosyncratic volatility discount is stronger for growth firms and why the value effect is stronger for high volatility firms. The reason is the idiosyncratic volatility hedging channel. An increase in aggregate volatility signals a recession, when expected risk premium is high. However, higher aggregate volatility also means higher idiosyncratic volatility, which diminishes the increase in the risk premium for growth options. Higher idiosyncratic volatility also makes growth options more valuable. Both effects mute the drop in the firm value in recessions. The idiosyncratic volatility hedging channel is the strongest for high volatility, growth, and high volatility growth firms. The returns to these firms covary least negatively with changes in expected aggregate volatility. So, these firms have the lowest aggregate volatility risk.

The cross-sectional predictions of my model find strong support in the data. I find in portfolio sorts that the idiosyncratic volatility discount is much stronger for growth firms and absent for value firms. I confirm it by running Fama-MacBeth regressions of returns on firm characteristics. Adding the product of volatility and market-to-book changes the signs of idiosyncratic volatility and market-to-book. It suggests that the interaction between idiosyncratic volatility and growth options predicted by my model can potentially explain the idiosyncratic volatility discount and the value effect.

Controlling for idiosyncratic volatility greatly increases the magnitude and significance of the size effect. The size effect predicts that small firms, which are also high volatility firms, earn high returns. The idiosyncratic volatility discount predicts just the opposite. My results suggest that the size effect seems weak because of this conflict, not because size per se is not important.

In time-series tests, I introduce the aggregate volatility risk factor - the BVIX factor. It is similar to the one used by Ang, Hodrick, Xing, and Zhang (2006). The BVIX factor is an arbitrage portfolio long in the firms with the most negative and short in the firms with the most positive return sensitivity to aggregate volatility increases. I show that BVIX earns a large premium controlling for the three Fama-French risk factors and is priced for different portfolio sets.

I show that high volatility, growth, and especially high volatility growth firms have large and negative BVIX betas. It means that they provide a hedge against the aggregate volatility risk, as predicted by my model. I also show that these three types of firms have significantly lower conditional CAPM betas in recessions than in expansions. The conditional betas provide additional evidence that high volatility, growth, and high volatility growth firms are a hedge against adverse economy-wide shocks. Their risk exposure turns out to be the lowest when the risk is the highest.

Augmenting the CAPM by the BVIX factor perfectly explains the idiosyncratic volatility discount and its dependence on market-to-book, as well as the abysmal returns to the highest volatility growth firms. The Intertemporal CAPM with BVIX also reduces by about a third the abnormally large value effect for high volatility firms.

Bali and Cakici (2007) claim that the idiosyncratic volatility discount is not robust to reasonable changes in the research design. I revisit their findings and show that their most decisive results suffer from selection bias. When Bali and Cakici (2007) look at NYSE only firms, they use the current listing instead of the listing at the portfolio formation date. The selection bias is more severe for high volatility firms, which are more likely to perform very well or very poorly and move between exchanges. I find that Bali and Cakici (2007) overestimate the return to the highest volatility NYSE firms by more than 50 bp per month. It explains why they do not find any difference between the returns to the highest and lowest volatility firms in what they call the NYSE only sample.

I test the competing behavioral explanations of the idiosyncratic volatility discount and find that they do not subsume my results. There is some evidence that the idiosyncratic volatility discount and the interaction of volatility and market-to-book are stronger if limits to arbitrage are higher. However, in covariance-based tests the link between the idiosyncratic volatility discount and limits to arbitrage seems to be explained by the three Fama-French risk factors, which contradicts the behavioral stories.

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8 Appendix. Proofs

This Appendix collects the proofs of the propositions in text. Some prepositions refer to the simulations described in the web appendix at

http : //outside2.simon.rochester.edu/phdresumes/barinov_alexander/simulations.pdf

Proposition 1. The value of the firm is given by

$$dV_t/V_t = (r+\pi_B-(\pi_B-\pi_S\Phi(d_1)\frac{S_t}{P_t})\cdot\frac{P_t}{V_t})dt+\Phi(d_1)\frac{S_t}{V_t}(\sigma_S dW_S+\sigma_I dW_I)+\sigma_B\frac{B_t}{V_t}dW_B \quad (25)$$

where

$$d_1 = \frac{log(S/K) + (r + \sigma_C^2/2 + \sigma_I^2/2)(T-t)}{\sqrt{(\sigma_C^2 + \sigma_I^2)\cdot(T-t)}} \quad (26)$$

The expected rate of return to the firm (the drift in the firm value, μ_V) decreases in idiosyncratic risk, σ_I, and increases in the value of assets in place, B.

Proof: Black and Scholes (1973) formula in my case yields

$$P_t = S_t\Phi(d_1) - \exp(r(T-t))K\Phi(d_2) \quad (27)$$

where $\Phi(\cdot)$ is the normal cdf, d_1 is as defined in (26), and $d_2 = d_1 - \widetilde{\sigma}$.

Applying the Ito's lemma and the no-arbitrage condition to the value of the firm, $V_t = P_t + B_t$, I find that the value of the firm follows

$$dV_t/V_t = (r + \pi_S\cdot\Phi(d_1)\frac{S_t}{V_t} + \pi_B\frac{B_t}{V_t})dt + \Phi(d_1)\frac{S_t}{V_t}(\sigma_S dW_S + \sigma_I dW_I) + \sigma_B\frac{B_t}{V_t}dW_B \quad (28)$$

Then I rearrange the expression for the drift

$$\mu_V = r + \pi_S\Phi(d_1)\frac{S_t}{V_t} + \pi_B\frac{B_t}{V_t} = r + \pi_B - [\pi_B - \pi_S\Phi(d_1)\frac{S_t}{P_t}]\cdot\frac{P_t}{V_t} \quad (29)$$

Determining the sign of the drift's derivatives with respect to idiosyncratic risk and assets in place is now simple and intuitive. The term in the square brackets is positive if assets in place earn higher returns than growth options, which is a sufficient condition to derive the value effect. The changes in assets in place, B_t, influence only the denominator of the last term in (29). As B_t increases, V_t increases as well, and the whole last term decreases if $(\pi_B - \pi_S\Phi(d_1)S_t/P_t) > 0$, meaning that an increase in B_t causes an increase in expected returns. Algebraically,

$$\frac{\partial\mu_V}{\partial B} = \frac{P_t}{V_t^2}\cdot(\pi_B - \pi_S\Phi(d_1)\frac{S_t}{P_t}) > 0, \quad (30)$$

An increase in idiosyncratic risk, σ_I, increases the price of growth options, P_t, and their fraction in the value of the firm, P_t/V_t. An increase in idiosyncratic risk also leads to a decrease in the option elasticity with respect to the price of the underlying asset, $\Phi(d_1)S_t/P_t$, (see Galai and Masulis, 1976, for a proof). Therefore, both parts of the last term in (29) increase as idiosyncratic risk increases, and expected returns decrease. Algebraically,

$$\frac{\partial \mu_V}{\partial \omega} = \pi_S \frac{\partial(\Phi(d_1)S_t/P_t)}{\partial \omega} \cdot \frac{P_t}{V_t} - (\pi_B - \pi_S \Phi(d_1)\frac{S_t}{P_t}) \cdot \frac{B_t}{V_t^2} \cdot \frac{\partial P_t}{\partial \omega} < 0 \; , \tag{31}$$

where the first term captures the effect of idiosyncratic risk on the option elasticity, and the second term captures the increase in the relative weight of growth options.

QED

Corollary 1. Define $IVar$ as the variance of the part of the return generating process (6), which is orthogonal to the pricing kernel. Then the idiosyncratic variance $IVar$ is

$$\begin{aligned} IVar = \sigma_S^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} \cdot (1 - \rho_{S\Lambda}^2) + \sigma_B^2 \cdot \frac{B^2}{V^2} \cdot (1 - \rho_{B\Lambda}^2) + \\ + \sigma_I^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} + \sigma_S \cdot \sigma_B \cdot \Phi(d_1) \cdot \frac{S}{V} \cdot \frac{B}{V} \cdot (\rho_{SB} - \rho_{B\Lambda} \cdot \rho_{S\Lambda}) \end{aligned} \tag{32}$$

I show that for all reasonable parameter values σ_I

$$\frac{\partial IVar}{\partial \sigma_I} > 0, \tag{33}$$

which implies that my empirical measure of idiosyncratic volatility - the standard deviation of Fama-French model residuals - is a noisy but valid proxy for σ_I.

Proof: The orthogonal to dW_Λ part of any diffusion is $dW_\bullet - \rho_{\bullet\Lambda} \cdot dW_\Lambda$. Therefore, (32) can be rewritten as

$$\begin{aligned} dV_t/V_t = (r + \pi_B - (\pi_B - \pi_S \Phi(d_1)\frac{S_t}{P_t}) \cdot \frac{P_t}{V_t})dt + \\ +[\Phi(d_1)\frac{S_t}{V_t} \cdot (\sigma_S(dW_S - \rho_{S\Lambda} dW_\Lambda) + \sigma_I dW_I) + \sigma_B \frac{B_t}{V_t} \cdot \\ \cdot(dW_B - \rho_{B\Lambda} dW_\Lambda)] + [\sigma_S \Phi(d_1)\frac{S_t}{V_t} \cdot \rho_{S\Lambda} + \sigma_B \frac{B_t}{V_t} \cdot \rho_{B\Lambda}] dW_\Lambda \end{aligned} \tag{34}$$

where the first square bracket contains the part orthogonal to dW_Λ and the second square bracket contains the part driven by dW_Λ. The standard deviation of the first square bracket is the model measure of idiosyncratic volatility, and its most natural empirical estimate is the standard deviation of an asset-pricing model's residuals (in the empirical part I choose the Fama-French model).

Applying Fubini's theorem and collecting terms yields, as claimed in Corollary 1, that the idiosyncratic variance is

$$\begin{aligned} IVar = \sigma_S^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} \cdot (1 - \rho_{S\Lambda}^2) + \sigma_B^2 \cdot \frac{B^2}{V^2} \cdot (1 - \rho_{B\Lambda}^2) + \\ + \sigma_I^2 \cdot \Phi^2(d_1) \cdot \frac{S^2}{V^2} + \sigma_S \cdot \sigma_B \cdot \Phi(d_1) \cdot \frac{S}{V} \cdot \frac{B}{V} \cdot (\rho_{SB} - \rho_{B\Lambda} \cdot \rho_{S\Lambda}) \end{aligned} \quad (35)$$

The analytical expression for the derivative of $IVar$ wrt σ_I is complicated, and its sign cannot be determined without simulations. The simulations (see the web appendix) show that at all empirically plausible parameter values the idiosyncratic volatility increases with the idiosyncratic risk parameter σ_I. The idiosyncratic volatility is also impacted by other parameters, so it is a noisy, but valid proxy for σ_I.

QED.

Proposition 2. The effect of idiosyncratic risk on returns, $\left|\frac{\partial \mu_V}{\partial \sigma_I}\right|$, is increasing in the value of assets in place B.

Proof:

$$\frac{\partial^2 \mu_V}{\partial \sigma_I \partial B} = -\pi_S \frac{\partial(\Phi(d_1) S_t / P_t)}{\partial \sigma_I} \cdot \frac{P_t}{V_t^2} + (\pi_B - \pi_S \Phi(d_1) \frac{S_t}{P_t}) \cdot \frac{B - P}{V^3} > 0 \quad (36)$$

The first term is always positive, and the second term is positive if $B > P$ and negative otherwise. However, for small B the first term becomes relatively large. Simulations in the web appendix show that the derivative is positive except for the parameter value that imply total volatility of 70% per annum or more and and market-to-book higher than 5. The simulations also show that for these extreme parameter values the expected return is about the same as for the parameter values yielding the positive derivative.

QED

Proposition 3 The elasticity of the risk premium in my model decreases (increases in the absolute magnitude) as idiosyncratic volatility increases:

$$\frac{\partial}{\partial \sigma_I} (\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}) < 0 \quad (37)$$

The elasticity of the risk premium in my model increases (decreases in the absolute magnitude) as the value of assets in place increases:

$$\frac{\partial}{\partial B} (\frac{\partial \lambda_V}{\partial \sigma_I} \cdot \frac{\sigma_I}{\lambda_V}) > 0 \quad (38)$$

The second cross-derivative of the elasticity with respect to idiosyncratic volatility and assets in place is positive:

$$\frac{\partial^2}{\partial\sigma_I\partial B}(\frac{\partial\lambda_V}{\partial\sigma_I}\cdot\frac{\sigma_I}{\lambda_V}) > 0 \tag{39}$$

Proof: It turns out that the derivative in (38) is the easiest to sign:

$$\frac{\partial}{\partial B}(\frac{\partial\lambda_V}{\partial\sigma_I}\cdot\frac{\sigma_I}{\lambda_V}) = \frac{1}{\lambda_V^2}\cdot(\frac{\partial^2\lambda_V}{\partial\sigma_I\partial B}\cdot\sigma_I\cdot\lambda_V - \frac{\partial\lambda_V}{\partial\sigma_I}\cdot\frac{\partial\lambda_V}{\partial B}\cdot\sigma_I) > 0 \tag{40}$$

The derivative in the first term of (40) is positive at reasonable parameter values (see Proposition 2) and the derivatives in the second term of (40) are positive and negative, respectively (see Proposition 1). So, at reasonable parameter values (40) is a sum of two positive terms.

$$\begin{aligned}\frac{\partial}{\partial\sigma_I}(\frac{\partial\lambda_V}{\partial\sigma_I}\cdot\frac{\sigma_I}{\lambda_V}) &= \frac{1}{\lambda_V^2}\cdot((\frac{\partial^2\lambda_V}{\partial\sigma_I^2}\cdot\sigma_I + \frac{\partial\lambda_V}{\partial\sigma_I})\cdot\lambda_V - (\frac{\partial\lambda_V}{\partial\sigma_I})^2\cdot\sigma_I) = \\ &= \frac{1}{\lambda_V}\cdot(\frac{\partial^2\lambda_V}{\partial\sigma_I^2}\cdot\sigma_I + \frac{\partial\lambda_V}{\partial\sigma_I}(1 - \frac{\partial\lambda_V}{\partial\sigma_I}\cdot\frac{\sigma_I}{\lambda_V}))\end{aligned} \tag{41}$$

The first term in (41) has an ambiguous sign and the second term is always negative. Simulations in the web appendix show that the first term is positive but small for empirically plausible parameters, and the overall sign of (41) is negative.

Taking the cross-derivative (39) is tedious and, as in the previous case, there is no obvious way to sign it without simulations. The simulations in the web appendix show that at reasonable parameter values it is positive.

QED

Proposition 4 The elasticity of the firm value with respect to idiosyncratic volatility increases with idiosyncratic volatility:

$$\frac{\partial}{\partial\sigma_I}(\frac{\partial V}{\partial\sigma_I}\cdot\frac{\sigma_I}{V}) > 0 \tag{42}$$

The elasticity of the firm value decreases in the value of assets in place:

$$\frac{\partial}{\partial B}(\frac{\partial V}{\partial\sigma_I}\cdot\frac{\sigma_I}{V}) < 0 \tag{43}$$

The second cross-derivative of the elasticity with respect to idiosyncratic volatility and assets in place is negative:

$$\frac{\partial^2}{\partial\sigma_I\partial B}(\frac{\partial V}{\partial\sigma_I}\cdot\frac{\sigma_I}{V}) < 0 \tag{44}$$

Proof: It turns out that the derivative in (43) is the easiest to sign. The value of growth options increases in idiosyncratic volatility, and the effect of idiosyncratic volatility is weaker if assets in place take a larger share of the firm value. Algebraically, the elasticity is the firm value derivative with respect to idiosyncratic volatility scaled by the firm value. The derivative is always positive and does not depend on the value of assets in place[6]. The firm value increases in the value of assets in place, which makes the whole ratio (i.e., the elasticity) decrease in assets in place.

The derivatives in (42) and (44) are complicated. The simulations in the web appendix show that their values are always positive except for the extreme growth firms (in the model, the market-to-book higher than 5 and annual total volatility higher than 50% per annum). However, the elasticity for those firms is still much larger than the elasticity of most other firms.

[6]The fact that the call option value increases in volatility is widely known in finance. The respective derivative is called vega and equals to $P \cdot \exp(-r \cdot (T-t))\phi(d_1)\sqrt{T-t}$.

Table 1. Descriptive Statistics

The table presents descriptive statistics for the idiosyncratic volatility quintiles. Idiosyncratic volatility is defined as the standard deviation of residuals from the Fama-French model, fitted to the daily data for each month (at least 15 valid observations are required). The idiosyncratic volatility quintile portfolios are formed at the end of each calendar month based on the idiosyncratic volatility estimated in this month, and held for one month afterwards. Panel A shows the raw returns and the Fama-French alphas (both equal-weighted and value-weighted) to the idiosyncratic volatility quintile portfolios formed using CRSP quintile breakpoints. Panel B repeats the same for the portfolios formed using NYSE breakpoints. NYSE firms are defined as firms with `exchcd`=1, where `exchcd` is the listing indicator at the portfolio formation date from the CRSP events file. In Panel C I report the Fama-French factor betas the means of size and market-to-book for the portfolios from Panel B. Size is defined as shares outstanding times price from the CRSP monthly returns file. Market-to-book is defined as Compustat item #25 times Compustat item #199 divided by Compustat item #60 plus Compustat item #74. The returns and betas are measured in the month after the portfolio formation. Size and market-to-book are measured in the month of portfolio formation. The t-statistics reported use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from July 1963 to December 2006.

Panel A. Returns, CRSP Breakpoints

	Low	**IVol2**	**IVol3**	**IVol4**	**High**	**L-H**
Raw EW	1.140	1.411	1.451	1.298	0.934	0.206
t-stat	*6.12*	*6.16*	*5.23*	*3.77*	*2.16*	*0.63*
FF EW	0.055	0.180	0.151	-0.057	-0.545	0.600
t-stat	*0.66*	*2.40*	*2.41*	*-0.78*	*-3.24*	*2.99*
Raw VW	0.970	1.072	1.096	0.787	0.003	0.967
t-stat	*5.79*	*5.43*	*4.28*	*2.42*	*0.01*	*3.01*
FF VW	0.055	0.071	0.031	-0.356	-1.269	1.324
t-stat	*1.12*	*1.38*	*0.44*	*-3.50*	*-7.87*	*6.86*

Panel B. Returns, NYSE Breakpoints

	Low	IVol2	IVol3	IVol4	High	L-H
Raw EW	1.085	1.371	1.450	1.477	1.131	-0.046
t-stat	*5.89*	*6.26*	*5.96*	*5.20*	*2.96*	*-0.17*
FF EW	0.486	0.645	0.654	0.606	0.169	0.317
t-stat	*5.59*	*7.83*	*8.08*	*8.21*	*1.56*	*2.22*
Raw VW	0.980	1.087	1.067	1.154	0.688	0.292
t-stat	*5.90*	*5.73*	*4.91*	*4.48*	*2.06*	*1.16*
FF VW	0.077	0.099	0.014	0.049	-0.517	0.594
t-stat	*1.60*	*1.96*	*0.22*	*0.65*	*-4.92*	*4.34*

Panel C. Fama-French Betas, Size, and Market-to-Book, NYSE Breakpoints

	Low	IVol2	IVol3	IVol4	High	L-H
$\boldsymbol{\beta}_{MKT}$	0.874	1.025	1.118	1.196	1.266	-0.392
t-stat	*60.1*	*55.0*	*48.8*	*42.5*	*35.7*	*-8.57*
$\boldsymbol{\beta}_{SMB}$	-0.256	-0.148	0.021	0.268	0.815	-1.071
t-stat	*-11.4*	*-6.34*	*0.40*	*4.2*	*15.4*	*-18.1*
$\boldsymbol{\beta}_{HML}$	0.172	0.141	0.092	-0.009	-0.163	0.336
t-stat	*3.62*	*2.61*	*1.93*	*-0.14*	*-2.40*	*3.21*
Size	2511	1956	1175	598	176	2335
t-stat	*7.80*	*8.86*	*8.37*	*7.88*	*7.25*	*7.71*
M/B	2.733	2.514	2.547	2.700	3.789	-1.055
t-stat	*13.2*	*20.5*	*23.6*	*25.9*	*21.0*	*-4.97*

Table 2. Double Sorts: Fama-French Abnormal Returns

The table presents monthly Fama-French abnormal returns to the 25 idiosyncratic volatility - market-to-book portfolios, sorted independently using NYSE (`exchcd`=1) breakpoints. Idiosyncratic volatility is defined as the standard deviation of residuals from the Fama-French model, fitted to the daily data for each firm-month (at least 15 valid observations are required). The idiosyncratic volatility (market-to-book) portfolios are rebalanced monthly (annually). Market-to-book is defined as Compustat item #25 times Compustat item #199 divided by Compustat item #60 plus Compustat item #74. Panel A shows value-weighted returns and Panel B reports equal-weighted returns. The t-statistics reported use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from August 1963 to December 2006.

Panel A. Value-Weighted Returns

	Low	**IVol2**	**IVol3**	**IVol4**	**High**	**L-H**
Value	-0.001	0.068	0.172	0.080	-0.104	0.103
t-stat	*-0.01*	*0.44*	*1.07*	*0.47*	*-0.63*	*0.48*
MB 2	0.028	0.032	-0.145	-0.020	-0.293	0.321
t-stat	*0.25*	*0.26*	*-1.22*	*-0.13*	*-2.02*	*1.66*
MB 3	0.028	0.019	-0.058	-0.091	-0.435	0.462
t-stat	*0.25*	*0.16*	*-0.50*	*-0.66*	*-2.73*	*2.46*
MB 4	0.179	0.070	0.018	-0.100	-0.309	0.488
t-stat	*1.75*	*0.69*	*0.18*	*-0.85*	*-2.08*	*2.66*
Growth	0.273	0.299	0.191	0.249	-0.565	0.838
t-stat	*2.35*	*3.34*	*1.62*	*1.92*	*-3.89*	*3.92*
V-G	-0.275	-0.231	-0.019	-0.169	0.461	0.735
t(V-G)	*-1.50*	*-1.28*	*-0.10*	*-0.88*	*2.47*	*2.80*

Panel B. Equal-Weighted Returns

	Low	**IVol2**	**IVol3**	**IVol4**	**High**	**L-H**
Value	0.203	0.283	0.479	0.386	0.528	-0.325
t-stat	*1.63*	*3.02*	*4.85*	*3.64*	*3.54*	*-1.81*
MB 2	0.008	0.249	0.139	0.261	-0.053	0.061
t-stat	*0.09*	*2.71*	*1.50*	*2.81*	*-0.49*	*0.42*
MB 3	-0.039	0.110	0.180	0.179	-0.120	0.081
t-stat	*-0.38*	*1.14*	*1.99*	*2.25*	*-0.91*	*0.47*
MB 4	-0.064	0.098	0.117	0.115	-0.335	0.270
t-stat	*-0.61*	*1.00*	*1.26*	*1.35*	*-3.02*	*1.59*
Growth	-0.029	0.115	0.154	-0.010	-0.677	0.648
t-stat	*-0.24*	*0.90*	*1.48*	*-0.11*	*-5.01*	*3.34*
V-G	0.233	0.168	0.325	0.396	1.206	0.973
t(V-G)	*1.86*	*1.39*	*2.72*	*3.02*	*8.23*	*4.92*

Table 3. Fama-MacBeth Regressions

Panel B presents the results of firm-level Fama-MacBeth regressions run each month. The dependent variable is raw monthly return. The independent variables are the current-month market beta, the percentage rank of previous year market capitalization, the percentage rank of previous year market-to-book, the percentage rank of previous month idiosyncratic volatility. Idiosyncratic volatility is defined as the standard deviation of residuals from the Fama-French model, fitted to the daily data for each firm-month (at least 15 valid observations are required). Market-to-book is defined as Compustat item #25 times Compustat item #199 divided by Compustat item #60 plus Compustat item #74. The R-squared is the average R-squared across all months. The t-statistics reported use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from August 1963 to December 2006.

	1	**2**	**3**	**4**
Beta	0.317	0.345	0.347	0.348
t-stat	6.08	7.04	6.97	7.10
Size	-0.0102	-0.0149	-0.0158	-0.0156
t-stat	-2.75	-4.95	-5.08	-5.20
M/B	-0.0109	-0.0092	0.0034	0.0050
t-stat	-3.81	-3.71	1.29	1.90
IVol		-0.0098		0.0016
t-stat		-3.11		0.53
IVol*M/B			-0.00022	-0.00024
t-stat			-4.35	-6.31
R-sq	0.034	0.043	0.042	0.045
Adj R-sq	0.032	0.041	0.040	0.042

Table 4. Is the BVIX Factor Priced?

Panel A reports the value-weighted returns to the aggregate volatility sensitivity quintiles. The quintiles are sorted from the most negative to the most positive sensitivity in the previous month. The return sensitivity to aggregate volatility is measured separately for each firm-month by running stock excess returns on market excess returns and the VIX index change using daily data (at least 15 non-missing returns are required). The VIX index is from CBOE. It measures the implied volatility of the one-month S&P100 options. The sensitivity portfolios are rebalanced monthly and held for one month. The last column reports the difference in returns between the lowest and the highest sensitivity quintiles (the BVIX factor).

Panel B reports the GRS statistics for different portfolios sets - the 25 idiosyncratic volatility - market-to-book portfolios from Table 2, the 25 size - market-to-book portfolios from Fama and French (1992), and the 48 industry portfolios from Fama and French (1997). For the CAPM and the Fama-French model the GRS statistics test if all alphas are jointly zero. For the ICAPM with the BVIX factor, I test if all alphas are jointly zero and if all BVIX betas are jointly zero. The returns to all portfolio sets are equal-weighted. The t-statistics use the Newey-West (1987) correction for autocorrelation and heteroscedasticity. The sample period is from February 1986 to December 2006.

Panel A. Value-Weighted Returns to Volatility Sensitivity Quintiles

	VIX 1	**VIX 2**	**VIX 3**	**VIX 4**	**VIX 5**	**BVIX**
Raw	1.342	1.077	1.074	1.008	0.462	0.880
t-stat	3.92	4.28	4.43	3.54	1.19	4.15
CAPM	0.216	0.097	0.102	-0.027	-0.765	0.981
t-stat	1.62	1.20	1.24	-0.37	-4.21	4.20
FF	0.271	0.049	0.048	-0.048	-0.584	0.856
t-stat	1.90	0.73	0.75	-0.64	-3.66	3.71

Panel B. BVIX Factor Pricing for Different Portfolio Sets

25 IVol - M/B portfolios

	$\boldsymbol{\alpha}_{CAPM}$	$\boldsymbol{\alpha}_{FF}$	$\boldsymbol{\alpha}_{ICAPM}$	$\boldsymbol{\beta}_{BVIX}$
GRS	4.129	3.427	3.978	3.721
p-value	*0.000*	*0.000*	*0.000*	*0.000*

25 Size - M/B portfolios

	$\boldsymbol{\alpha}_{CAPM}$	$\boldsymbol{\alpha}_{FF}$	$\boldsymbol{\alpha}_{ICAPM}$	$\boldsymbol{\beta}_{BVIX}$
GRS	4.129	3.721	3.427	3.978
p-value	*0.000*	*0.000*	*0.000*	*0.000*

48 industry portfolios

	$\boldsymbol{\alpha}_{CAPM}$	$\boldsymbol{\alpha}_{FF}$	$\boldsymbol{\alpha}_{ICAPM}$	$\boldsymbol{\beta}_{BVIX}$
GRS	1.787	1.849	1.676	1.997
p-value	*0.003*	*0.002*	*0.008*	*0.001*

Table 5. Aggregate Volatility Risk Loadings

The table presents the BVIX betas of the 25 idiosyncratic volatility - market-to-book portfolios. The portfolios are sorted independently using NYSE (`exchcd`=1) breakpoints. The BVIX betas are estimated from the ICAPM with the BVIX factor. The BVIX factor is the difference in value-weighted returns between the quintiles of firms with the lowest and highest sensitivity of returns to the changes in the VIX index. The return sensitivity to changes in the VIX index is measured separately for each firm-month by running stock excess returns on market excess returns and the VIX change using daily data (at least 15 non-missing returns are required). The return sensitivity portfolios are formed at the end of each month based on this month return sensitivities and held for one month. The idiosyncratic volatility (market-to-book) portfolios are rebalanced monthly (annually). Panel A shows the results for value-weighted returns, and Panel B reports the results for equal-weighted returns. The t-statistics reported use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from January 1986 to December 2006.

Panel A. Value-Weighted Returns

	Low	**IVol2**	**IVol3**	**IVol4**	**High**	**L-H**
Value	0.186	0.127	0.066	0.230	-0.013	0.199
t-stat	*2.05*	*0.92*	*0.53*	*2.44*	*-0.14*	*1.99*
MB 2	0.059	0.089	0.104	0.061	-0.186	0.245
t-stat	*0.67*	*0.81*	*0.84*	*0.60*	*-1.42*	*1.19*
MB 3	0.057	0.186	0.125	0.037	-0.114	0.172
t-stat	*0.72*	*1.72*	*1.50*	*0.60*	*-1.59*	*1.54*
MB 4	0.172	0.295	0.194	-0.017	-0.201	0.374
t-stat	*2.40*	*3.98*	*2.31*	*-0.25*	*-3.21*	*3.39*
Growth	0.209	0.024	0.062	-0.030	-0.395	0.604
t-stat	*3.91*	*0.51*	*0.83*	*-0.44*	*-2.94*	*3.57*
V-G	-0.023	0.104	0.004	0.260	0.382	0.405
t(V-G)	*-0.26*	*0.74*	*0.03*	*2.32*	*2.75*	*3.37*

Panel B. Equal-Weighted Returns

	Low	**IVol2**	**IVol3**	**IVol4**	**High**	**L-H**
Value	0.061	0.051	0.022	0.012	-0.115	0.176
t-stat	*1.21*	*0.83*	*0.34*	*0.20*	*-0.68*	*1.11*
MB 2	0.031	0.066	0.038	0.002	-0.196	0.227
t-stat	*0.51*	*1.07*	*0.52*	*0.03*	*-1.38*	*1.37*
MB 3	0.082	0.074	0.050	-0.023	-0.257	0.340
t-stat	*1.28*	*1.09*	*0.72*	*-0.38*	*-1.56*	*1.73*
MB 4	0.041	0.101	0.036	-0.068	-0.348	0.390
t-stat	*0.68*	*1.71*	*0.61*	*-1.26*	*-2.26*	*2.28*
Growth	0.175	0.113	0.004	-0.160	-0.527	0.702
t-stat	*2.47*	*2.24*	*0.11*	*-2.20*	*-2.56*	*2.67*
V-G	-0.114	-0.063	0.017	0.172	0.412	0.526
t(V-G)	*-1.59*	*-1.39*	*0.36*	*2.17*	*4.01*	*3.57*

Table 6. Conditional CAPM Betas across Business Cycle

The table reports conditional CAPM betas across different states of the world for five arbitrage portfolios. IVol is the portfolio long in the low volatility quintile and short in the high volatility quintile. IVolh is long in low volatility growth portfolio and short in high volatility growth portfolio. HMLl (HMLh) is long in low (high) volatility value and short in low (high) volatility growth. IVol55 is long in high volatility growth portfolio and short in one-month Treasury bill. Recession (Expansion) is defined as the period when the expected market risk premium is higher (lower) than its in-sample mean. The expected risk premiums and the conditional betas are assumed to be linear functions of dividend yield, default spread, one-month Treasury bill rate, and term premium. The left part of the table presents the results with value-weighted returns, and the right part looks at equal-weighted returns. The standard errors reported use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from August 1963 to December 2006.

	Value-Weighted			**Equal-Weighted**		
	Rec	**Exp**	**Diff**	**Rec**	**Exp**	**Diff**
IVol	-0.563	-0.765	0.202	-0.641	-0.734	0.093
se	*0.034*	*0.037*	*0.051*	*0.033*	*0.032*	*0.047*
IVolh	-0.521	-0.809	0.288	-0.533	-0.811	0.278
se	*0.038*	*0.044*	*0.061*	*0.046*	*0.051*	*0.071*
HMLl	-0.005	-0.075	0.070	-0.270	-0.198	-0.072
se	*0.008*	*0.009*	*0.012*	*0.013*	*0.013*	*0.018*
HMLh	-0.123	-0.409	0.286	-0.223	-0.447	0.223
se	*0.019*	*0.028*	*0.035*	*0.020*	*0.028*	*0.036*
IVol55	1.467	1.644	-0.177	1.468	1.616	-0.147
se	*0.022*	*0.025*	*0.035*	*0.035*	*0.034*	*0.048*

Table 7. Explaining the Idiosyncratic Volatility Effects

The table reports monthly alphas of the four arbitrage portfolios (IVol, IVolh, HMLh, and IVol55) described in the heading of Table 6. The asset-pricing models I fit to their returns are the CAPM, the Fama-French model (FF), and the ICAPM with BVIX. The BVIX factor is defined in the heading of Table 4. In the conditional versions of the models the conditional betas are assumed to be linear functions of dividend yield, default spread, one-month Treasury bill rate, and term premium. Panel A and B report results for value- and equal-weighted returns, respectively. The standard errors reported use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from February 1986 to December 2006.

Panel A. Value-Weighted Returns

	Unconditional				**Conditional**		
	CAPM	**FF**	**ICAPM**		**CAPM**	**ICAPM**	
	$\boldsymbol{\alpha}_{CAPM}$	$\boldsymbol{\alpha}_{FF}$	$\boldsymbol{\alpha}_{ICAPM}$	$\boldsymbol{\beta}_{BVIX}$	$\boldsymbol{\alpha}_{CAPM}$	$\boldsymbol{\alpha}_{ICAPM}$	$\boldsymbol{\beta}_{BVIX}$
IVol	0.944	0.547	0.421	0.533	0.658	0.286	0.431
t-stat	*2.95*	*2.52*	*1.25*	*3.26*	*2.27*	*0.89*	*2.32*
IVolh	1.137	0.674	0.544	0.604	0.809	0.389	0.487
t-stat	*3.03*	*2.08*	*1.38*	*3.57*	*2.24*	*1.00*	*2.48*
HMLh	1.329	0.591	0.954	0.382	1.156	0.872	0.330
t-stat	*3.55*	*2.24*	*2.58*	*2.75*	*3.23*	*2.49*	*2.21*
IVol55	-0.831	-0.330	-0.443	-0.395	-0.645	-0.362	-0.328
t-stat	*-2.97*	*-1.55*	*-1.43*	*-2.94*	*-2.36*	*-1.20*	*-2.15*

Panel B. Equal-Weighted Returns

	Unconditional				**Conditional**		
	CAPM	**FF**	**ICAPM**		**CAPM**	**ICAPM**	
	$\boldsymbol{\alpha}_{CAPM}$	$\boldsymbol{\alpha}_{FF}$	$\boldsymbol{\alpha}_{ICAPM}$	$\boldsymbol{\beta}_{BVIX}$	$\boldsymbol{\alpha}_{CAPM}$	$\boldsymbol{\alpha}_{ICAPM}$	$\boldsymbol{\beta}_{BVIX}$
IVol	0.486	0.129	0.115	0.378	0.266	0.014	0.292
t-stat	*1.44*	*0.57*	*0.30*	*2.05*	*0.83*	*0.04*	*1.38*
IVolh	0.958	0.391	0.269	0.702	0.607	0.119	0.567
t-stat	*2.16*	*1.40*	*0.53*	*2.67*	*1.46*	*0.24*	*1.91*
HMLh	2.135	1.612	1.730	0.412	1.939	1.630	0.358
t-stat	*6.90*	*6.90*	*5.95*	*4.01*	*6.26*	*5.48*	*3.20*
IVol55	-0.951	-0.595	-0.434	-0.527	-0.749	-0.362	-0.449
t-stat	*-2.46*	*-2.82*	*-0.96*	*-2.56*	*-1.89*	*-0.78*	*-1.94*

Table 8. Robustness: Revisiting Bali and Cakici (2007)

In this table I look at equal-weighted Fama-French alphas of idiosyncratic volatility quintiles formed using NYSE only firms. Panel A uses the daily measure of idiosyncratic volatility, and Panel B uses the monthly measure. Idiosyncratic volatility is the standard deviation of Fama-French residuals. For the daily measure, in each firm-month with at least 15 valid observations I fit the model to daily returns. For the monthly measure, I fit the model to monthly returns over the previous 60 months (at least 24 valid observations required). I first classify firms as NYSE using the current listing, `hexcd` from the CRSP returns file, to mimic Bali and Cakici (2007). Then I add the delisting returns, and then use the listing at the portfolio formation date, `exchcd` from the CRSP events file. The t-statistics reported use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from August 1963 to December 2004.

Panel A. Daily Volatility, NYSE Only

	Low	**IVol2**	**IVol3**	**IVol4**	**High**	**L-H**
$\textbf{Raw}_{\texttt{hexcd}}$	1.162	1.404	1.539	1.614	1.415	-0.253
t-stat	*6.21*	*6.32*	*6.09*	*5.42*	*3.86*	*-1.08*
$\alpha_{\texttt{hexcd}}$	0.060	0.182	0.225	0.181	-0.260	0.319
t-stat	*0.86*	*2.49*	*2.62*	*1.95*	*-2.20*	*2.67*
$\alpha_{\texttt{+Delist}}$	0.063	0.183	0.227	0.182	-0.286	0.349
t-stat	*0.91*	*2.50*	*2.64*	*1.96*	*-2.42*	*2.91*
$\alpha_{\texttt{exchcd}}$	0.000	0.113	0.099	0.007	-0.850	0.849
t-stat	*-0.01*	*1.66*	*1.23*	*0.08*	*-6.89*	*6.30*

Panel B. Monthly Volatility, NYSE Only

	Low	**IVol2**	**IVol3**	**IVol4**	**High**	**L-H**
$\textbf{Raw}_{\texttt{hexcd}}$	1.164	1.324	1.435	1.467	1.672	-0.508
t-stat	*6.72*	*6.22*	*5.61*	*4.87*	*4.45*	*-1.87*
$\alpha_{\texttt{hexcd}}$	0.079	0.111	0.072	-0.001	0.045	0.034
t-stat	*1.14*	*1.58*	*0.86*	*-0.01*	*0.38*	*0.27*
$\alpha_{\texttt{+Delist}}$	0.080	0.112	0.076	-0.003	-0.057	0.137
t-stat	*1.16*	*1.60*	*0.91*	*-0.03*	*-0.47*	*1.07*
$\alpha_{\texttt{exchcd}}$	0.063	0.049	0.004	-0.134	-0.605	0.668
t-stat	*0.91*	*0.72*	*0.05*	*-1.44*	*-5.00*	*4.87*

Table 9. Behavioral Stories: Characteristic-Based Tests

The table presents the results of firm-level Fama-MacBeth regressions run each month. The dependent variable is raw return. RInst is the percentage rank of previous quarter residual institutional ownership less 100. Residual institutional ownership is defined as the residual from the logistic regression (19) of institutional ownership on log size and its square. Short is the percentage rank of probability to be on special defined in (20) and (21). All other variables are described in the heading of Table 2. The sample excludes all stocks below the 20th NYSE/AMEX size percentile at the date of the institutional ownership measurement. The t-statistics reported use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from April 1980 to December 2006.

Panel A. Residual Institutional Ownership

	1	**2**	**3**	**4**	**5**
Beta	0.271	0.279	0.280	0.281	0.280
t-stat	*4.50*	*4.67*	*4.71*	*4.74*	*4.71*
Size	-0.0113	-0.0111	-0.0116	-0.0122	-0.0127
t-stat	*-2.72*	*-2.68*	*-2.80*	*-2.96*	*-3.06*
MB	-0.0076	-0.0066	-0.0062	0.0060	0.0054
t-stat	*-2.13*	*-1.92*	*-1.79*	*1.70*	*1.56*
IVol	-0.0173	-0.0166	-0.0040	0.0081	0.0006
t-stat	*-3.93*	*-3.84*	*-1.02*	*2.00*	*0.17*
RInst		0.0051	-0.0081	-0.0069	-0.0063
t-stat		*3.43*	*-4.88*	*-4.58*	*-4.14*
IVol*			0.00027	0.00024	0.00009
***RInst**			*5.77*	*5.52*	*1.61*
IVol*				-0.00025	-0.00012
***MB**				*-4.58*	*-1.87*
IVol*					0.000003
MB					*4.21*
***Rinst**					

Panel B. Probability on Special

	1	**2**	**3**	**4**	**5**
Beta	0.290	0.295	0.298	0.300	0.300
t-stat	*4.49*	*4.62*	*4.66*	*4.68*	*4.70*
Size	-0.0122	-0.0293	-0.0282	-0.0273	-0.0275
t-stat	*-2.79*	*-5.38*	*-5.19*	*-5.10*	*-5.09*
MB	-0.0070	-0.0038	-0.0030	0.0107	0.0086
t-stat	*-2.02*	*-1.14*	*-0.92*	*2.71*	*2.27*
IVol	-0.0166	-0.0140	0.0025	0.0166	0.0075
t-stat	*-3.75*	*-3.48*	*0.57*	*3.24*	*1.20*
Short		-0.0137	0.0014	0.0016	0.0012
t-stat		*-4.30*	*0.43*	*0.49*	*0.36*
IVol*			-0.00032	-0.00029	-0.00017
***Short**			*-4.51*	*-4.23*	*-1.74*
IVol*				-0.00028	-0.00014
***MB**				*-4.57*	*-1.44*
IVol*					-0.000002
MB					*-1.92*
***Short**					

Table 10. Behavioral Stories: Covariance-Based Tests

The table presents the idiosyncratic volatility discount across the limits-to-arbitrage quintiles formed using NYSE (`exchcd`=1) breakpoints. RI is residual institutional ownership, defined as the residual from the logistic regression (19) of institutional ownership on log size and its square. Sh is the probability to be on special, defined in (20) and (21). The idiosyncratic volatility discount is defined as the difference in value-weighted abnormal returns between extreme idiosyncratic volatility quintiles. I form the quintiles using NYSE (`exchcd`=1) breakpoints. The abnormal returns are from the CAPM, the Fama-French model (FF), and the ICAPM with BVIX. For the ICAPM, I also report the BVIX betas. The BVIX factor is defined in the heading of Table 4. The t-statistics reported use Newey-West (1987) correction for heteroscedasticity and autocorrelation. The sample period is from February 1986 to December 2006.

Panel A. Residual Institutional Ownership

	Low	**RI 2**	**RI 3**	**RI 4**	**High**	**L-H**
$\boldsymbol{\alpha}_{CAPM}$	1.364	0.982	0.456	0.372	0.570	0.795
t-stat	*3.11*	*2.53*	*1.48*	*1.24*	*1.54*	*1.83*
$\boldsymbol{\alpha}_{FF}$	0.722	0.479	0.069	-0.006	0.424	0.299
t-stat	*2.23*	*1.55*	*0.28*	*-0.03*	*1.40*	*0.77*
$\boldsymbol{\alpha}_{ICAPM}$	0.807	0.532	0.018	-0.156	0.064	0.744
t-stat	*2.00*	*1.36*	*0.05*	*-0.48*	*0.15*	*1.84*
$\boldsymbol{\beta}_{BVIX}$	0.568	0.459	0.447	0.537	0.516	0.052
t-stat	*2.47*	*2.65*	*3.23*	*4.27*	*3.73*	*0.35*

Panel B. Probability on Special

	Low	**Sh 2**	**Sh 3**	**Sh 4**	**High**	**H-L**
$\boldsymbol{\alpha}_{CAPM}$	0.370	0.726	0.891	0.854	1.054	0.684
t-stat	*1.14*	*2.07*	*2.36*	*1.87*	*2.07*	*1.69*
$\boldsymbol{\alpha}_{FF}$	0.039	0.316	0.325	0.204	0.151	0.111
t-stat	*0.14*	*1.15*	*1.21*	*0.61*	*0.49*	*0.32*
$\boldsymbol{\alpha}_{ICAPM}$	0.082	0.218	0.284	0.234	0.384	0.302
t-stat	*0.25*	*0.60*	*0.71*	*0.48*	*0.68*	*0.66*
$\boldsymbol{\beta}_{BVIX}$	0.294	0.517	0.618	0.631	0.683	0.389
t-stat	*3.23*	*3.49*	*3.08*	*2.90*	*2.36*	*1.52*